Computing on a (free energy) budget:

Reversible Computation

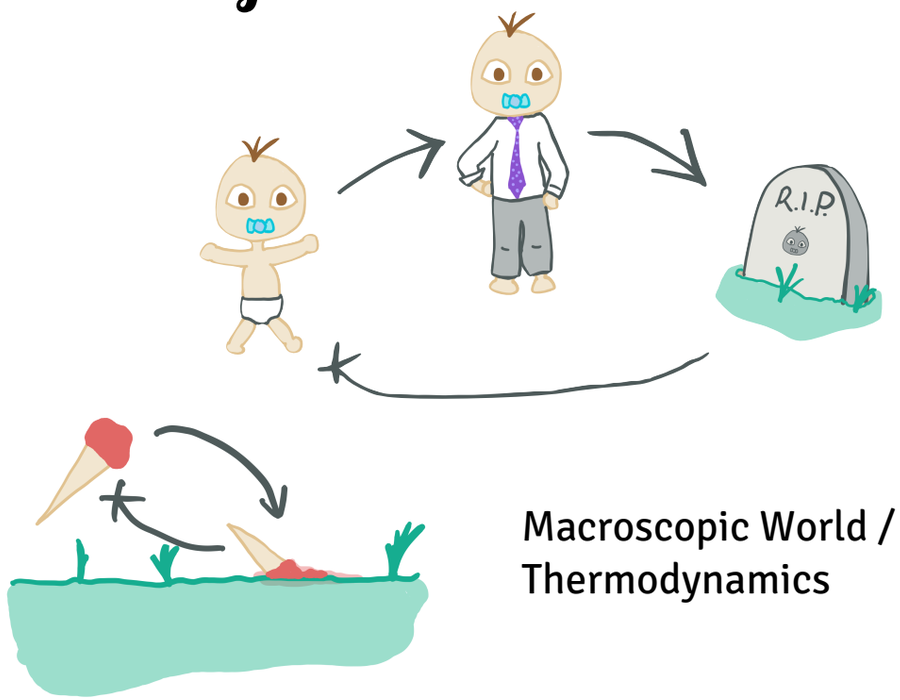
William Earley

Micklem Lab • DAMTP

What

Physics

Irreversible

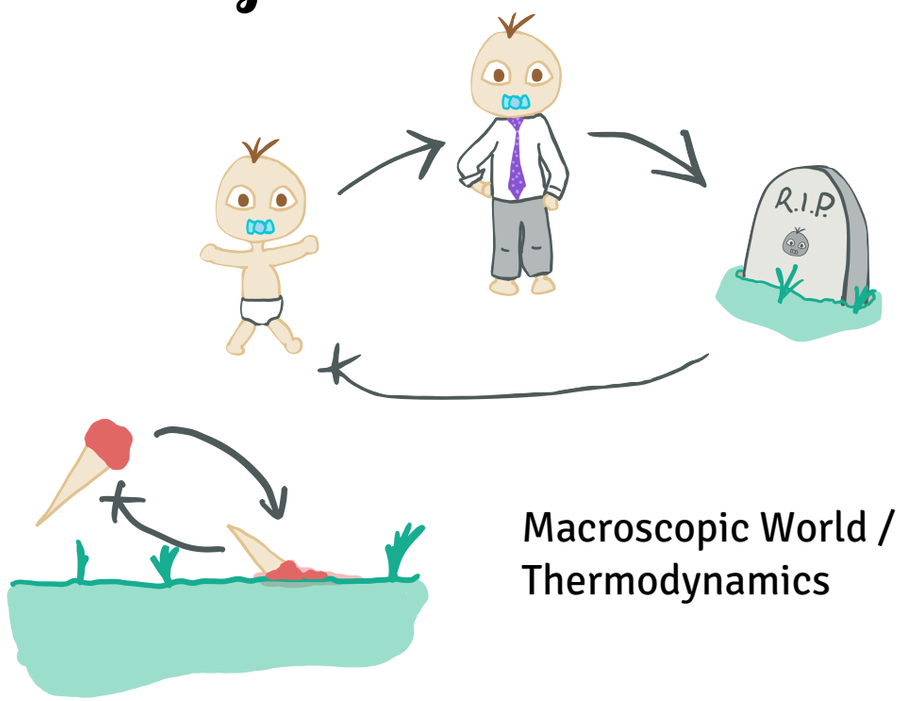


Macroscopic World /
Thermodynamics

What

Physics

Irreversible

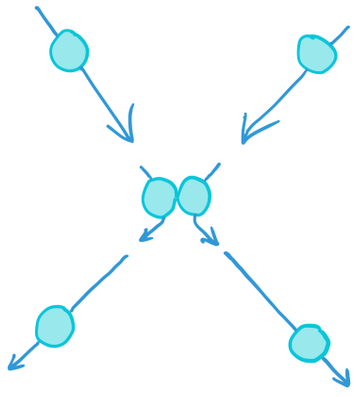


Macroscopic World / Thermodynamics

Reversible

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}\,|\psi(0)\rangle$$

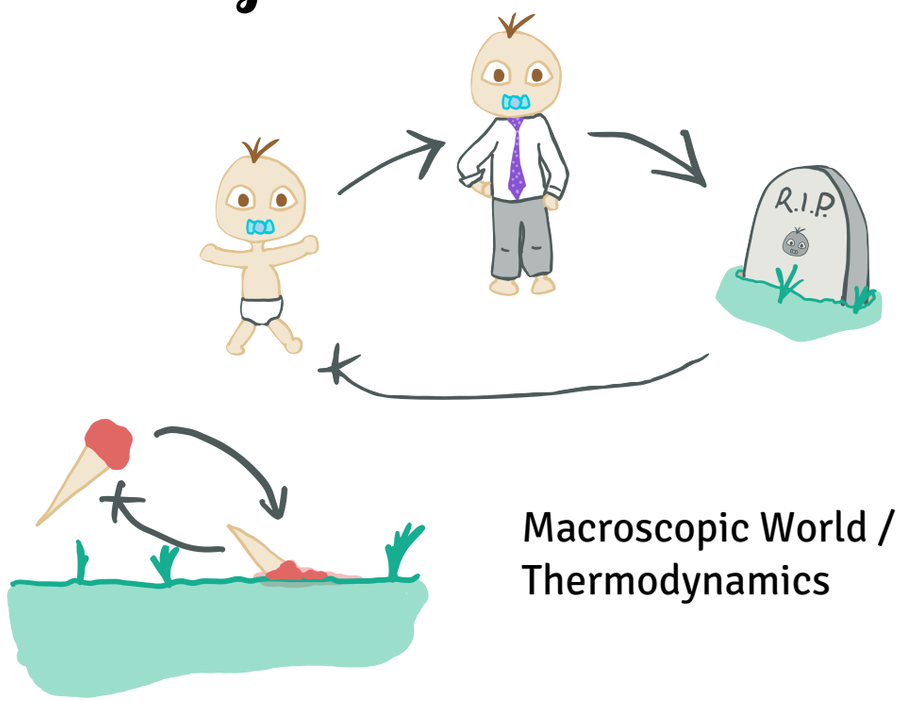
Microscopic Laws



What

Physics

Irreversible



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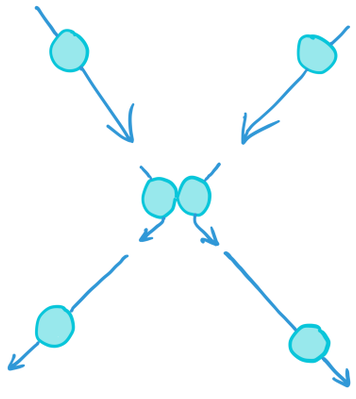


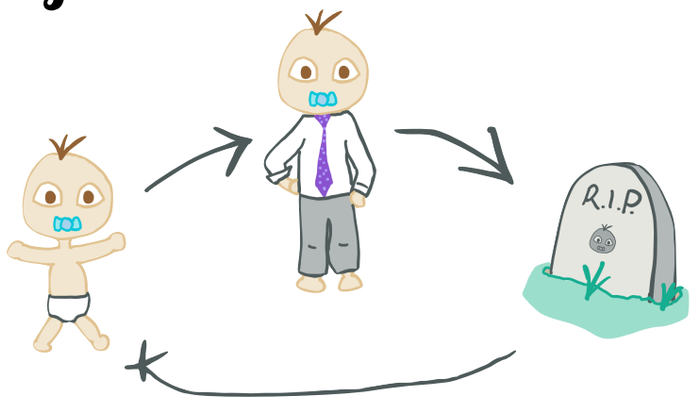
Laminar Flow[1]

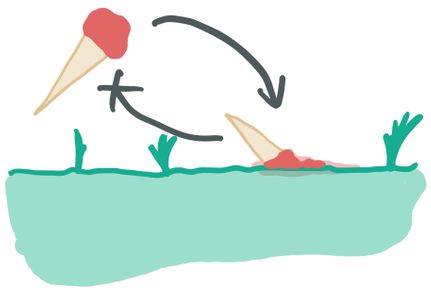
[1]John DeMoss and Kevin Cahill — 2007 — 'Laminar Flow' — University of New Mexico — Dept. Physics & Astronomy

What

Physics

Irreversible





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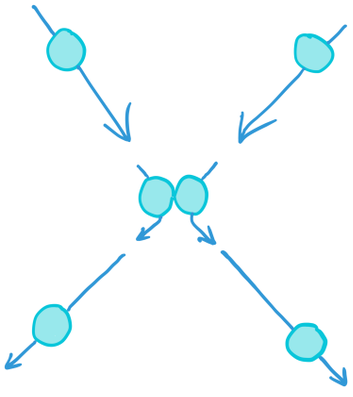
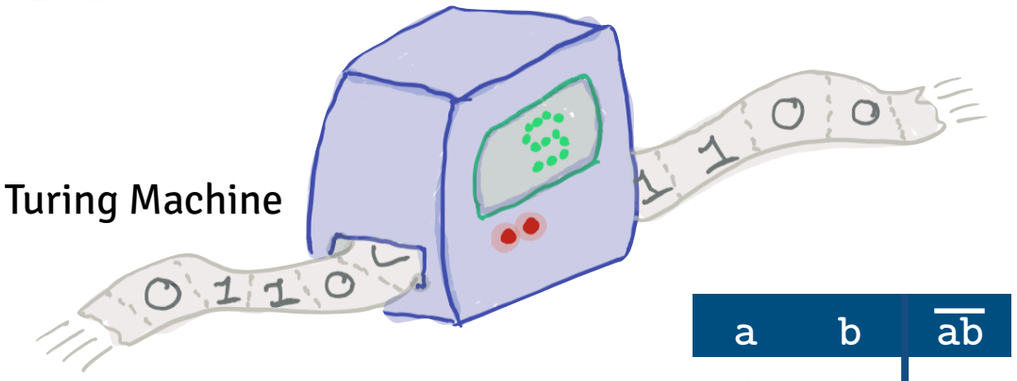


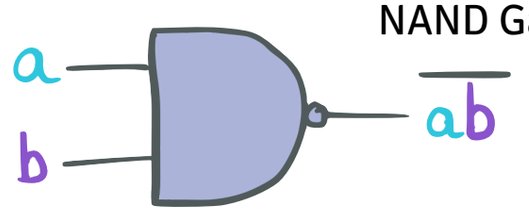
Laminar Flow[1]

Computing

Turing Machine



NAND Gate



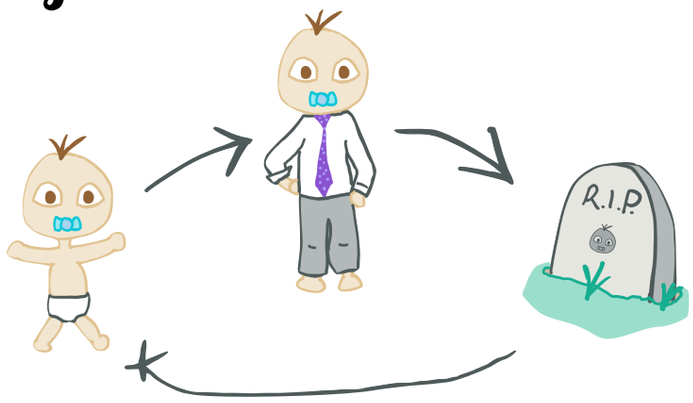
a	b	$\overline{ab}$
0	0	1
0	1	1
1	0	1
1	1	0

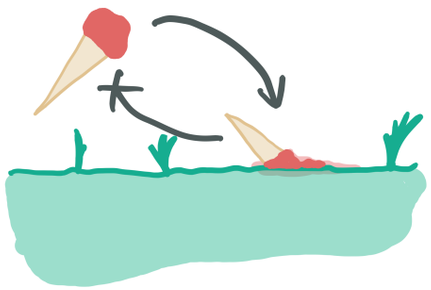
[1]John DeMoss and Kevin Cahill — 2007 — 'Laminar Flow' — University of New Mexico — Dept. Physics & Astronomy

What

Physics

Irreversible





Macroscopic World / Thermodynamics

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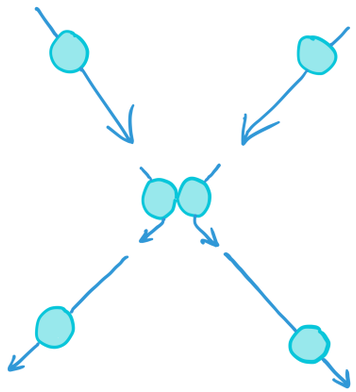
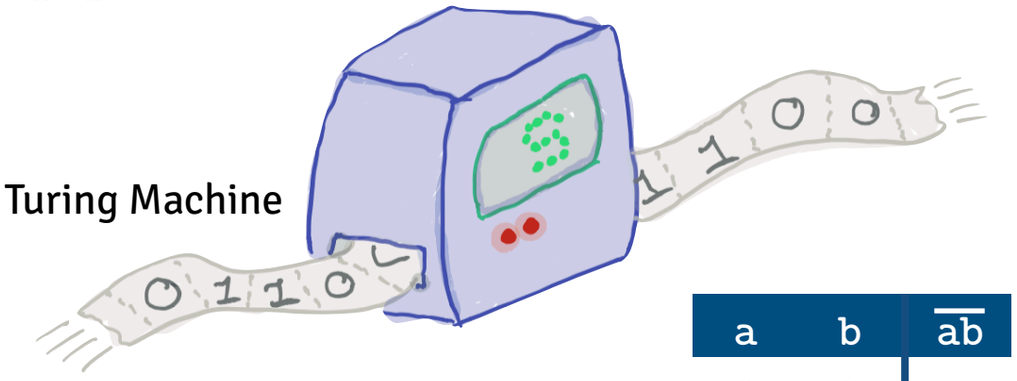


Laminar Flow[1]

Computing

Turing Machine



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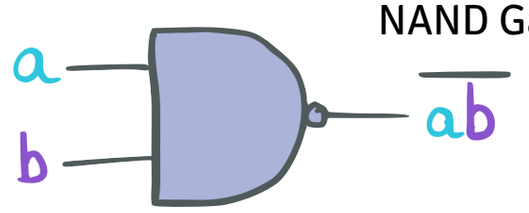
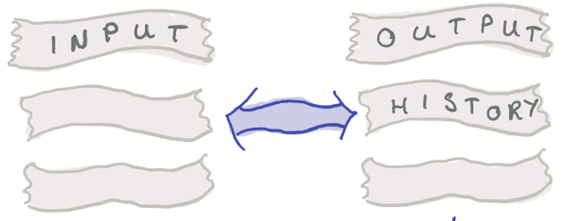
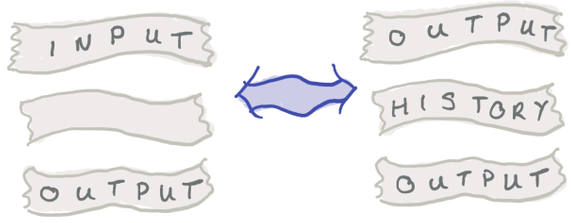
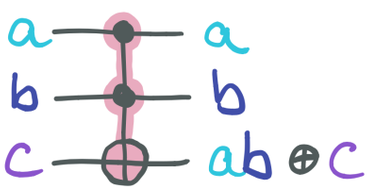


a	b	$\overline{ab}$
0	0	1
0	1	1
1	0	1
1	1	0



Bennett Embedding[2] in *Reversible* Turing Machine



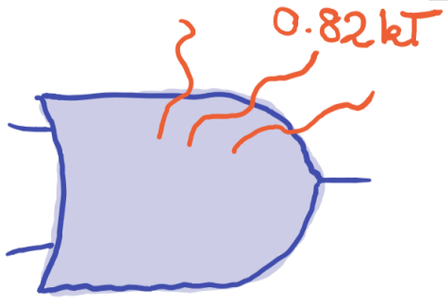


Toffoli Gate

a	b	c	a	b	ab⊕c
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

[1]John DeMoss and Kevin Cahill — 2007 — 'Laminar Flow' — University of New Mexico — Dept. Physics & Astronomy
[2]Charles H Bennett — 1973 — 'Logical Reversibility of Computation' — IBM J. Res. Dev.

Why



$$I = -\sum_i p_i \log p_i$$

$$\Delta q \geq k_B T \Delta I$$

Landauer[2]-Szilard[1] Principle

[1]Leo Szilard — 1929 — 'On the decrease of entropy in a thermodynamic system by the intervention of intelligent beings' — Zeitschrift für Physik
[2]Rolf Landauer — 1961 — 'Irreversibility and heat generation in the computing process' — IBM J. Res. Dev.

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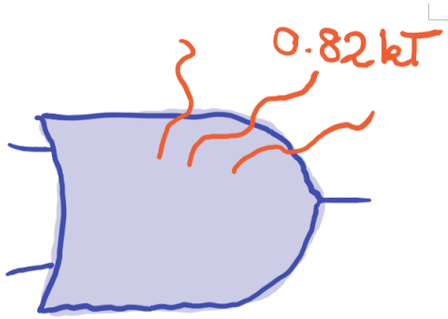

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End of Moore's Law[3]

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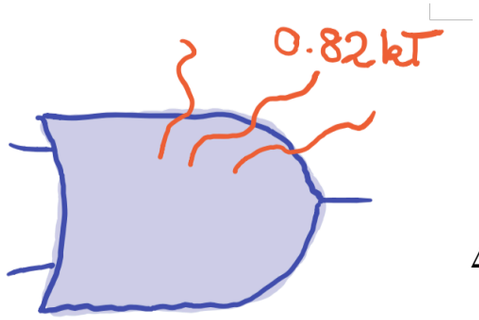

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Scaling

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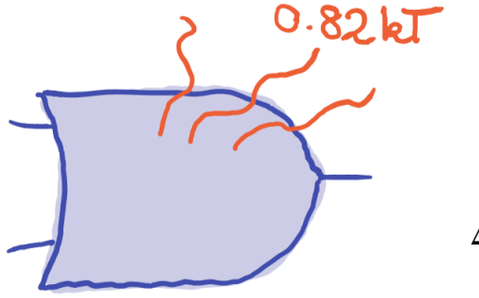

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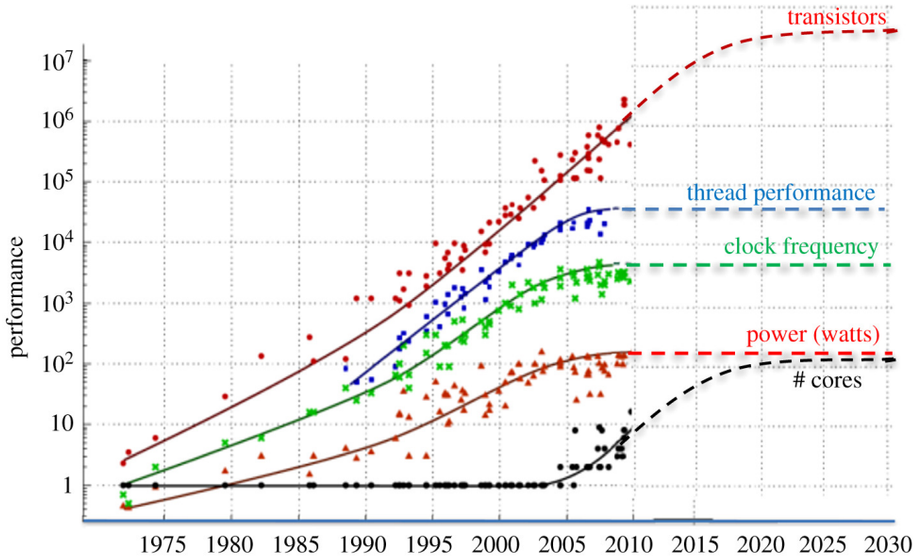

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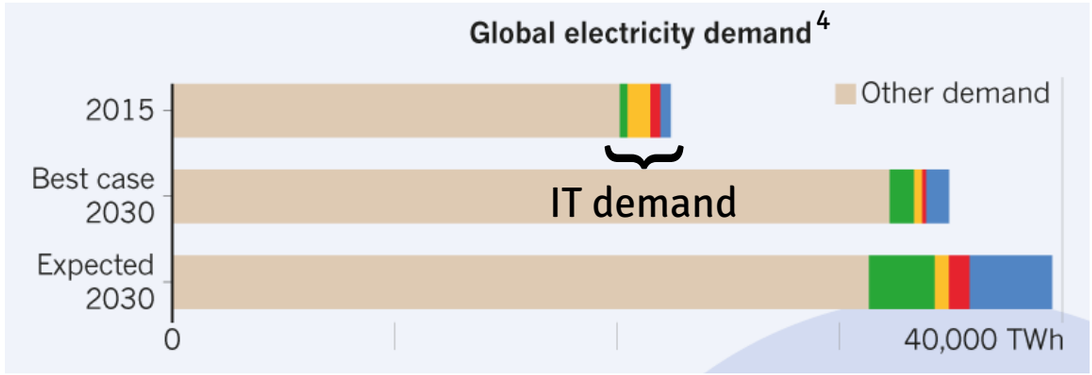

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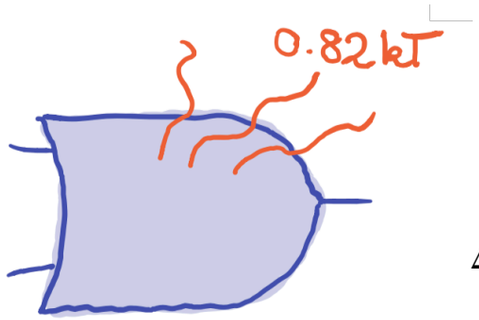

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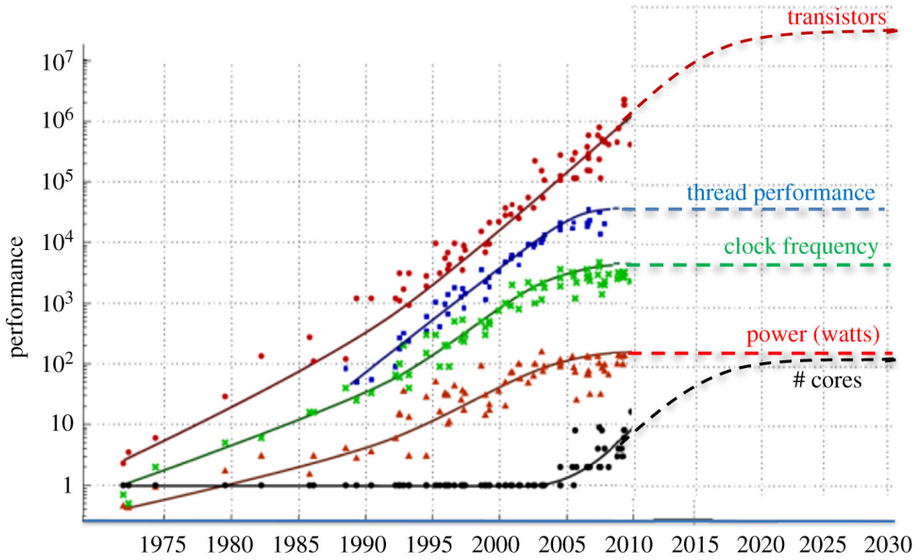

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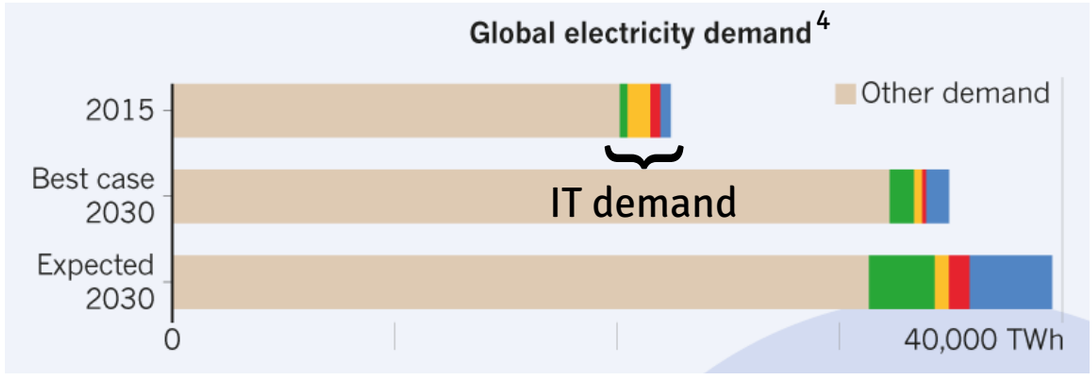

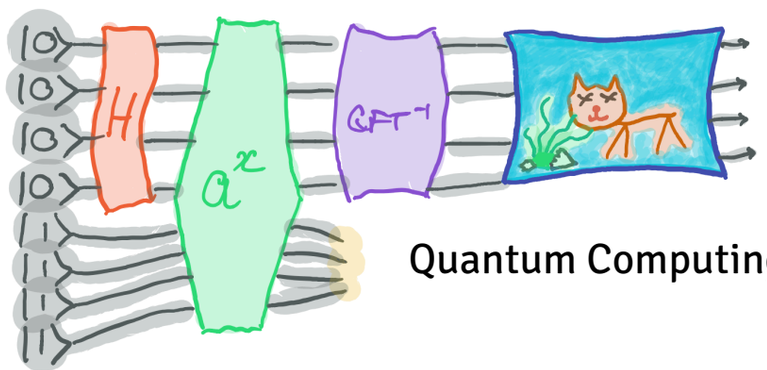

Quantum Computing

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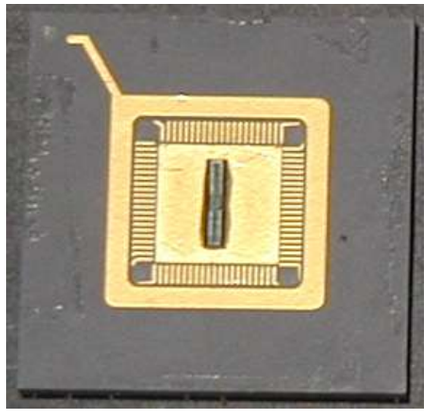
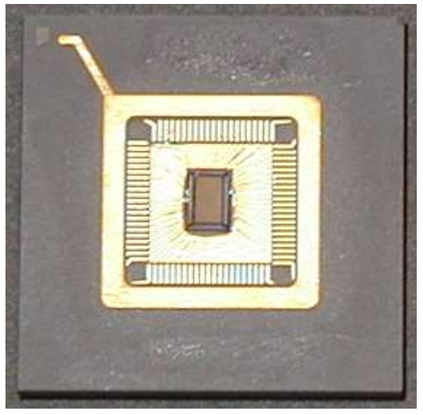
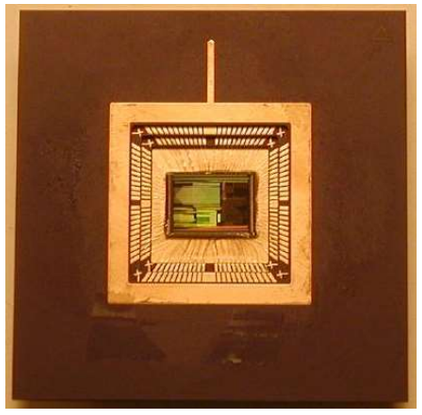
When



Tick[1]



FlatTop[1]



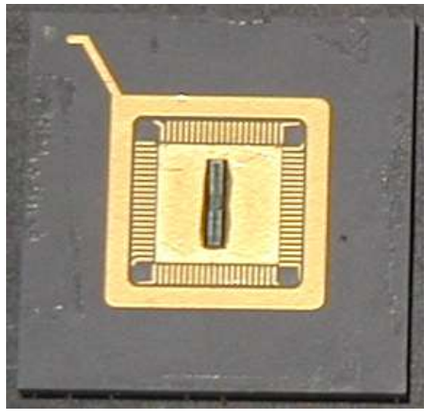
Pendulum[1]

Silicon:

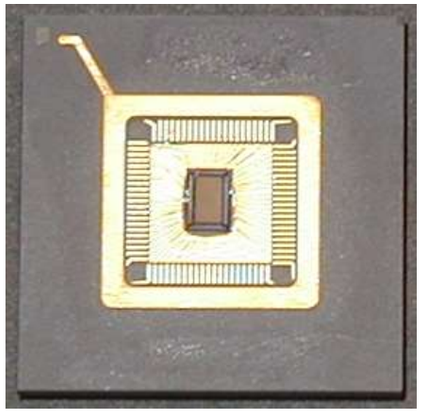
Yesterday?

[1]Frank, Ammer, Love, Rixner, Vieri — 1996–1999 — Tick, FlatTop, Pendulum processors

When



Tick[1]



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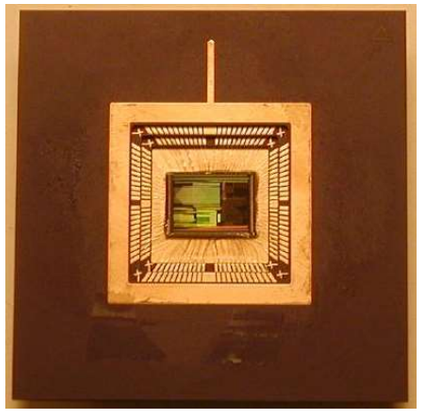

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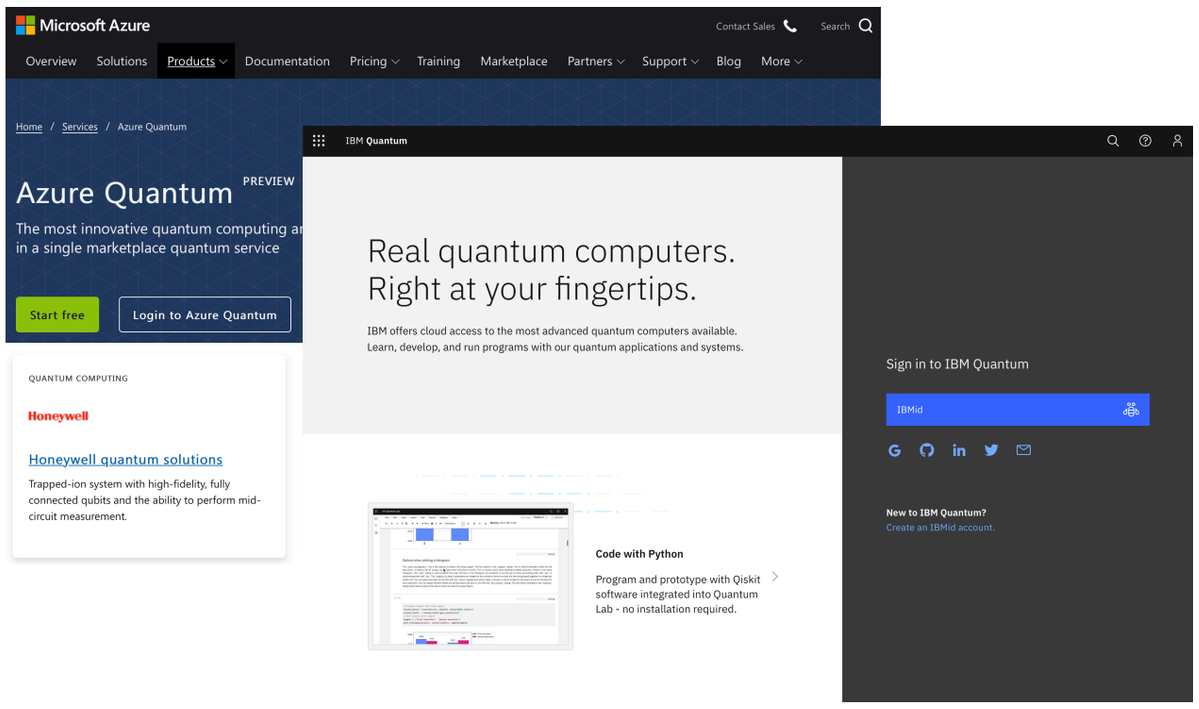

Quantum:

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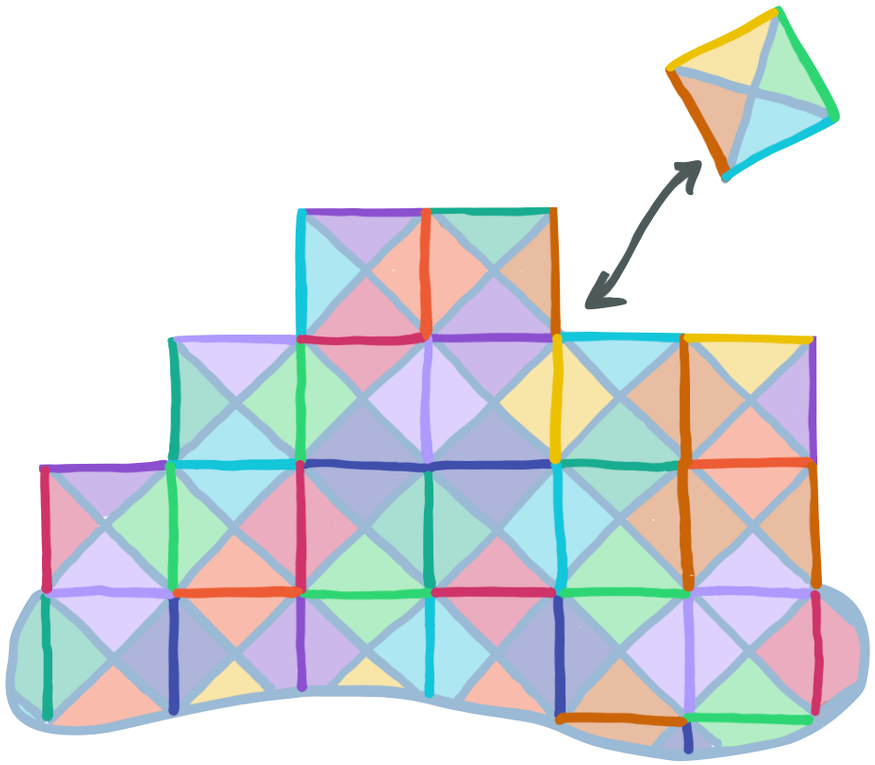
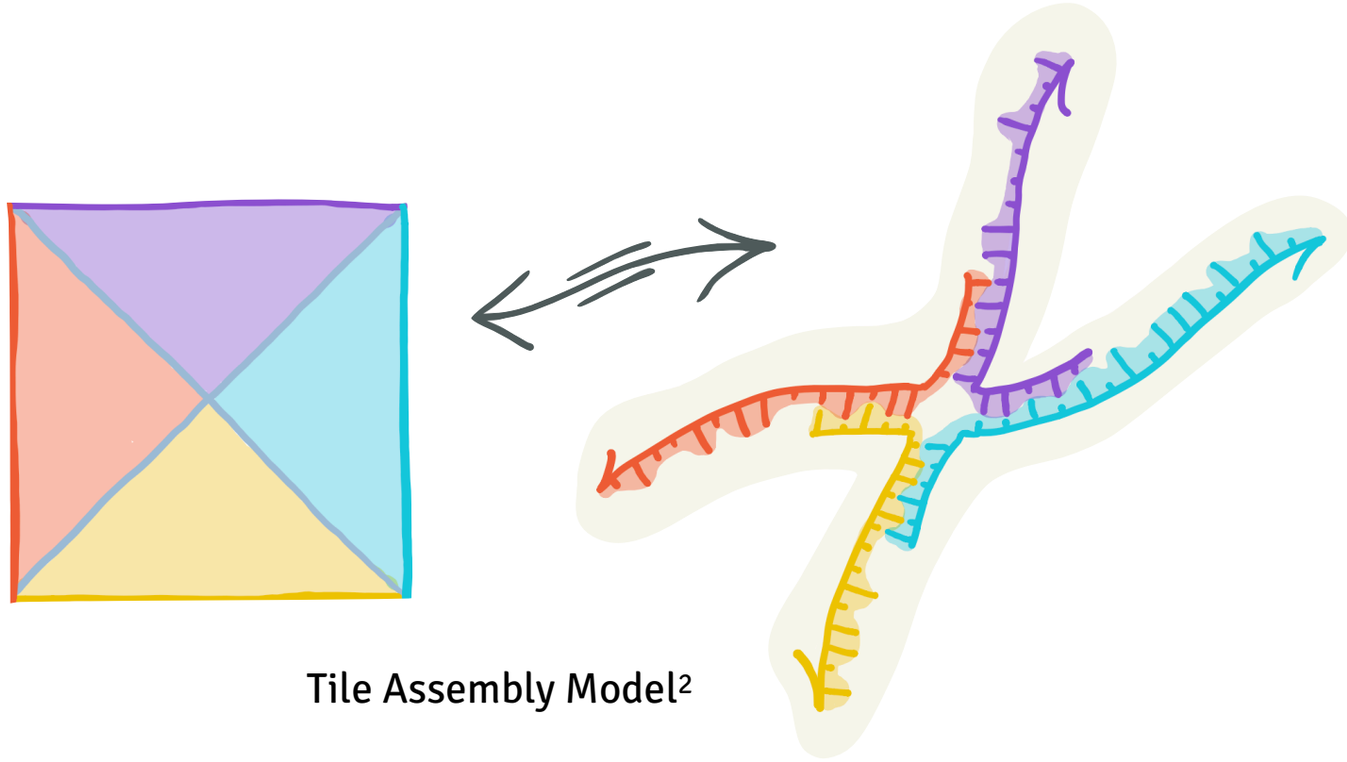


Wang Tiles[1]

Molecular:

Soon?

Reversible Surface Tiles[3][4]



Tile Assembly Model[2]

[1]Hao Wang — 1961 — 'Proving theorems by pattern recognition—II' — Bell Labs Tech. J.
[2]Erik Winfree — 'Simulations of Computing by Self-Assembly' — 1998 — Tech. rep. California Institute of Technology
[3]Brailovskaya T, Gowri G, Yu S, Winfree E — 2019 — 'Reversible Computation Using Swap Reactions on a Surface' — DNA Comp. and Mol. Prog.
[4]Samuel Clamons, Lulu Qian, Erik Winfree — 2020 — 'Programming and simulating chemical reaction networks on a surface' — J. R. Soc. Interface

How

$$3 + 4 = 7$$

How

$$3 + 4 = 7$$

$$7 = ? + ?$$

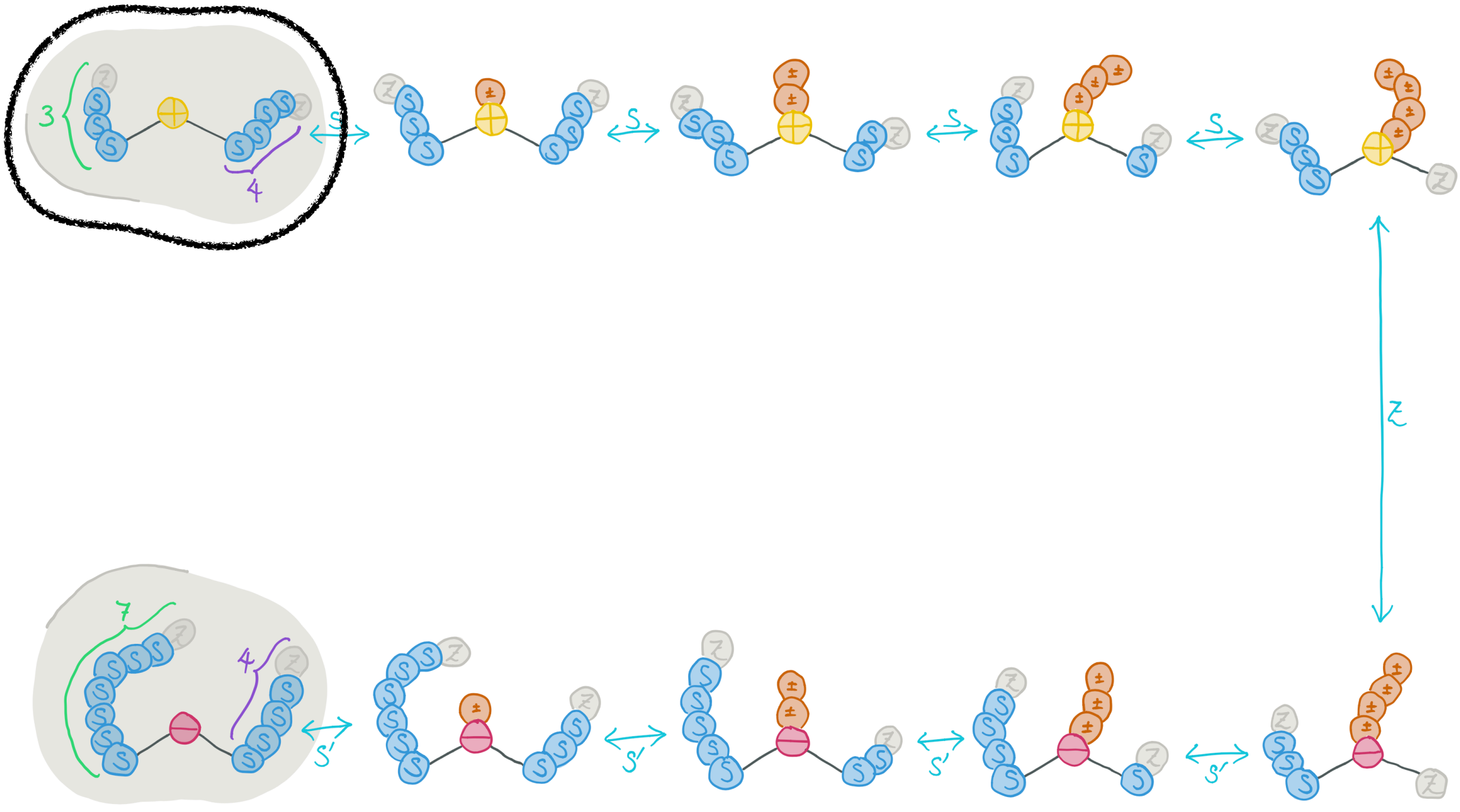
How

$$3 + 4 = 7$$

$$7 = ? + ?$$

$$3 + 4 = 7 \bar{+} 4$$

The א (Aleph) Calculus



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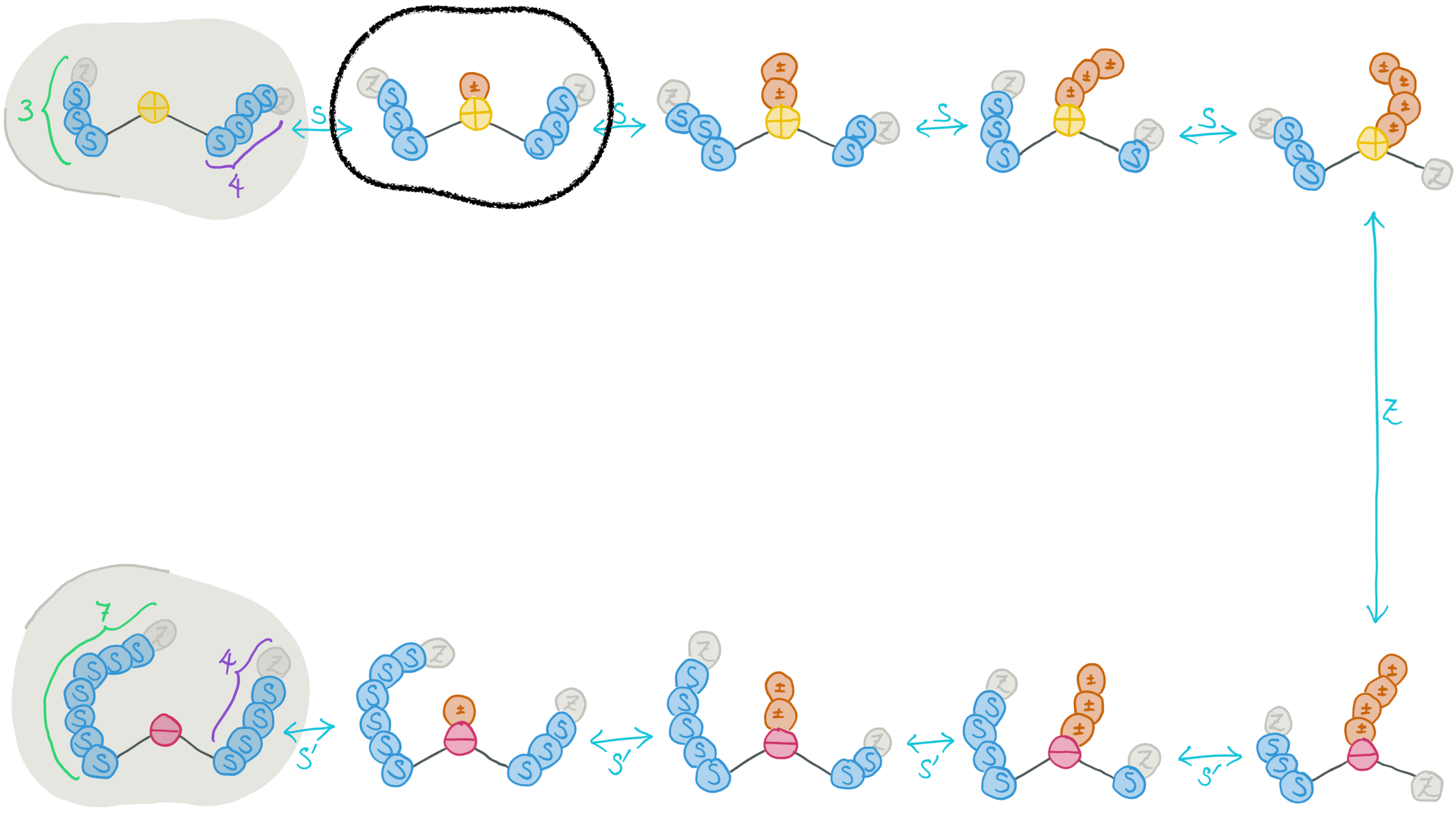
The א (Aleph) Calculus



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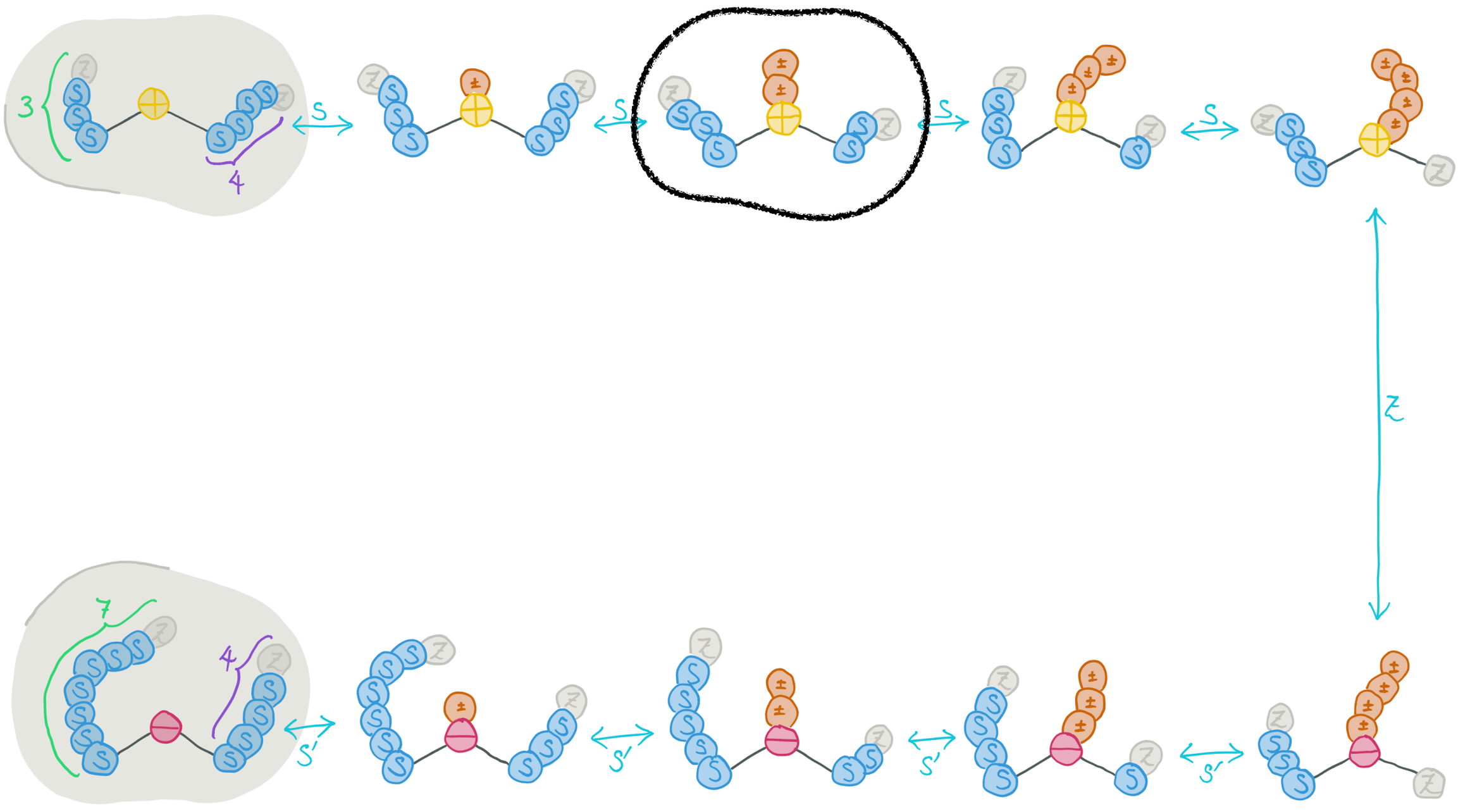

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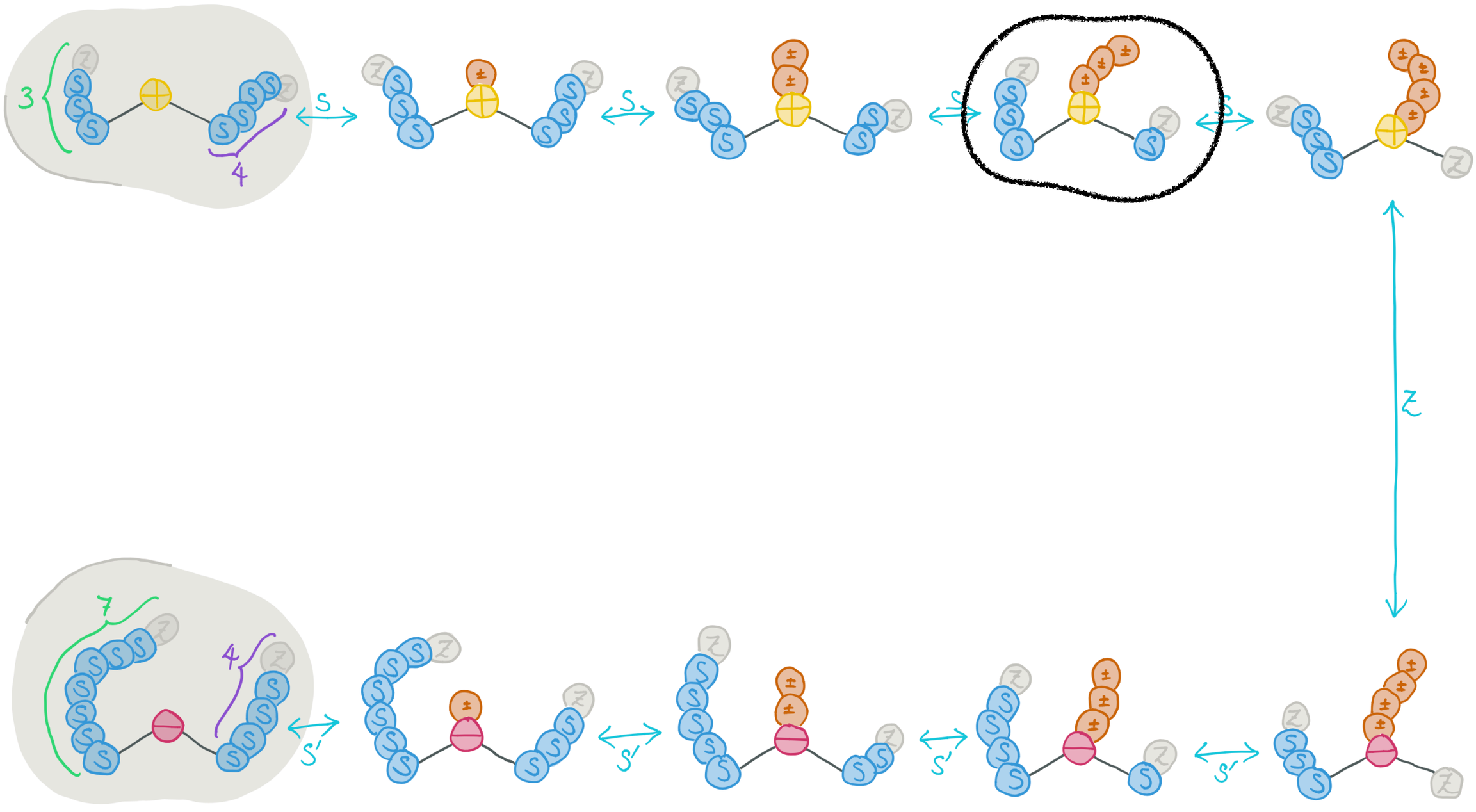

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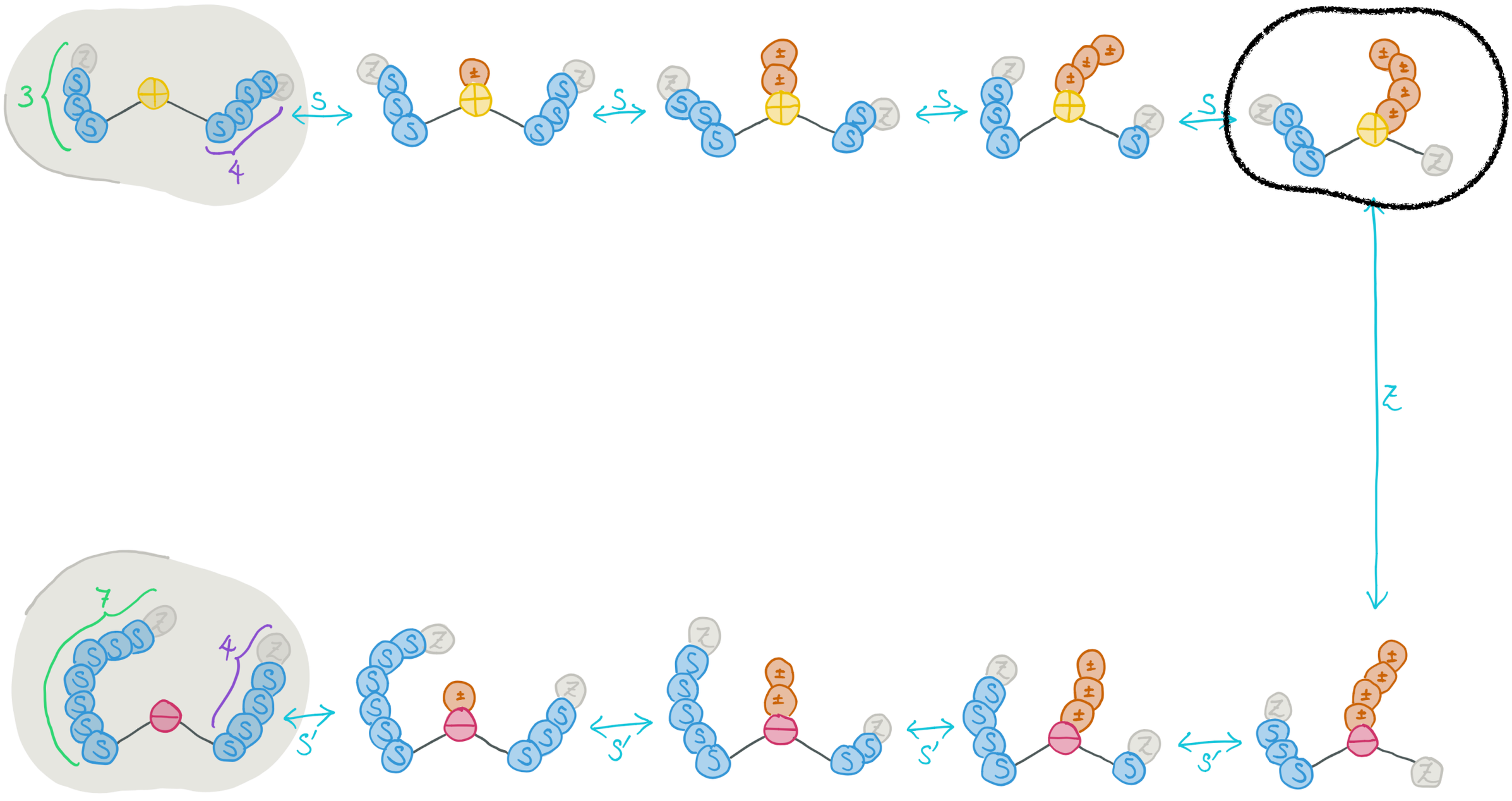

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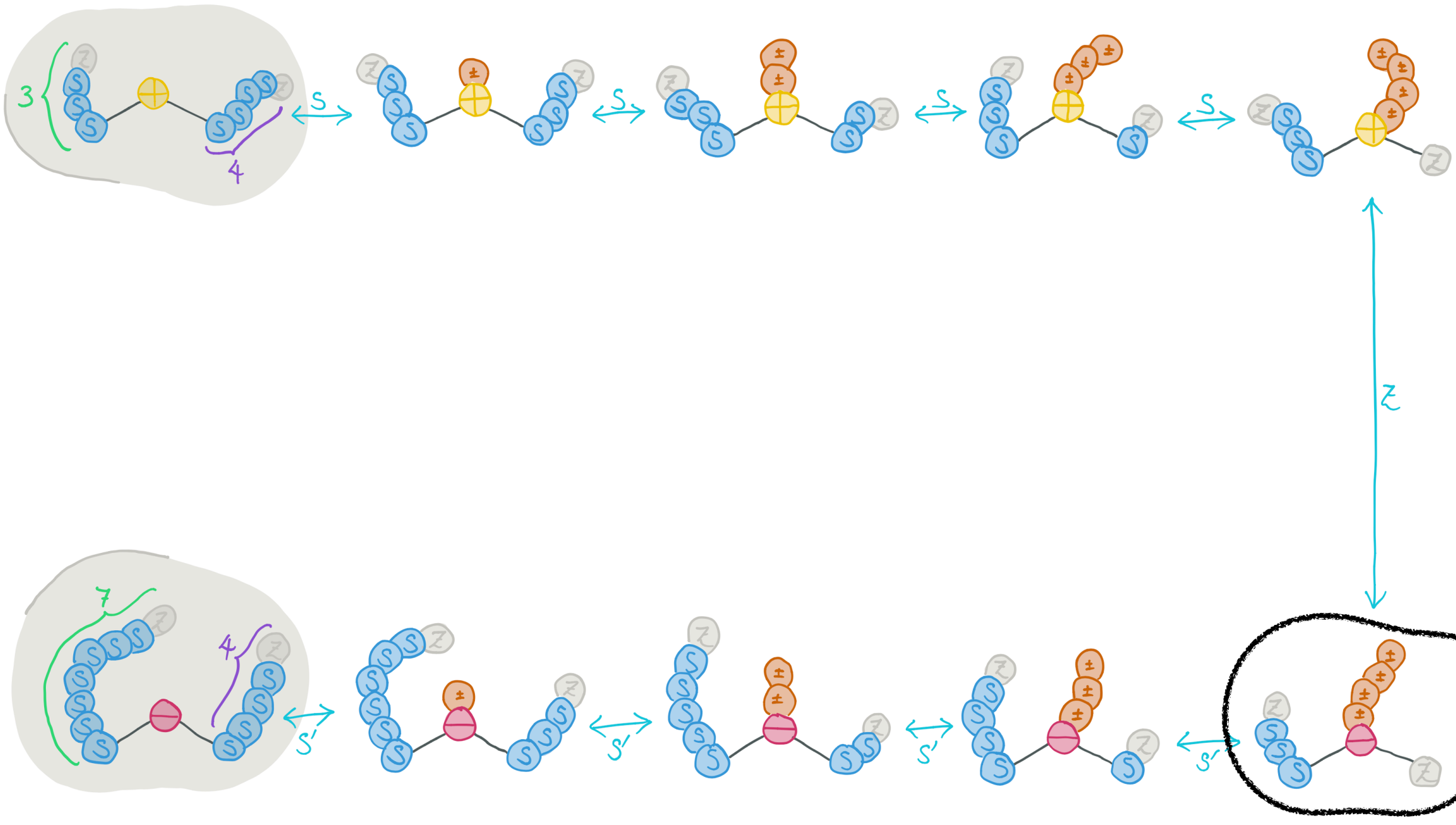
The א (Aleph) Calculus



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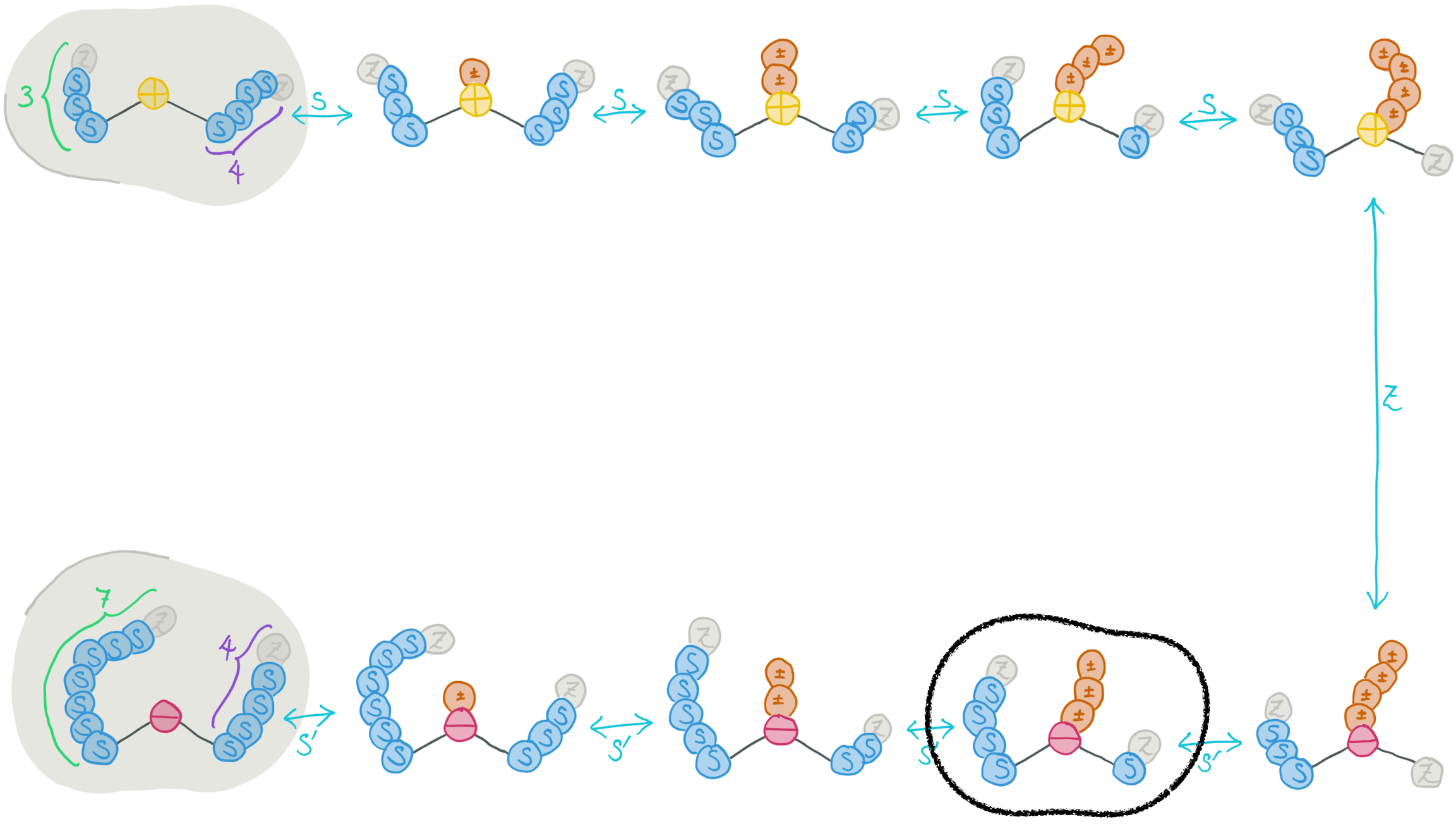

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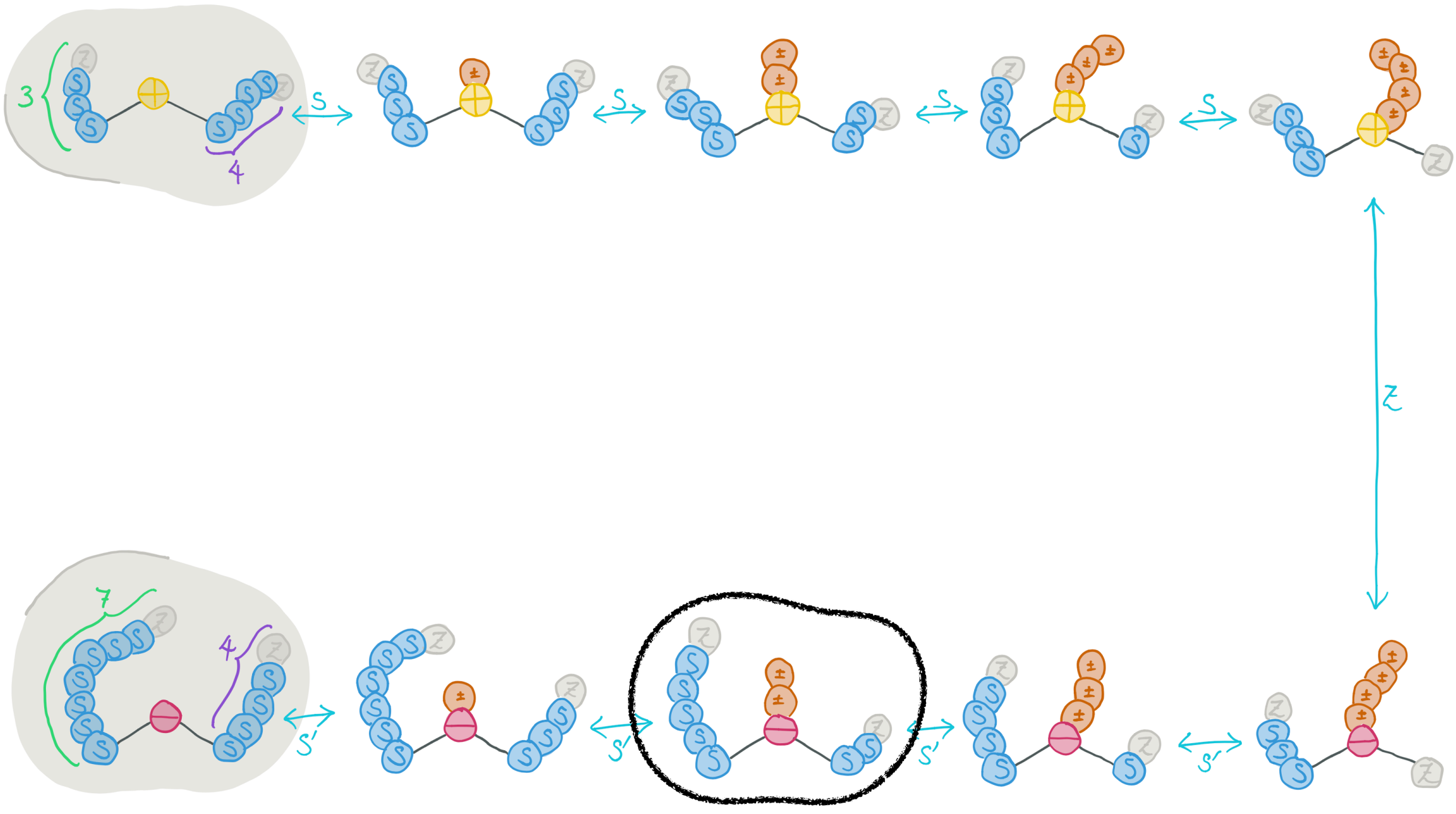

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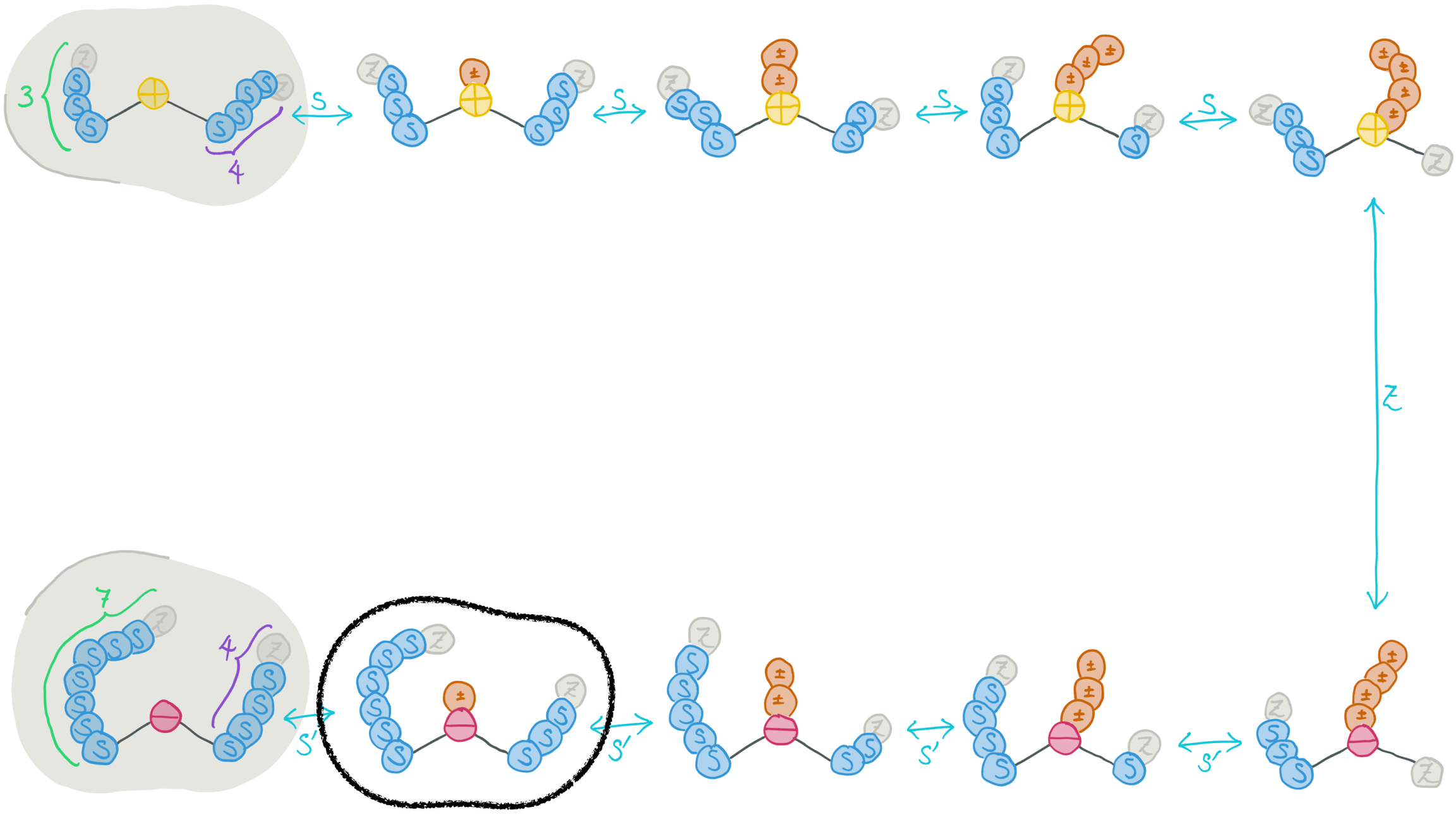

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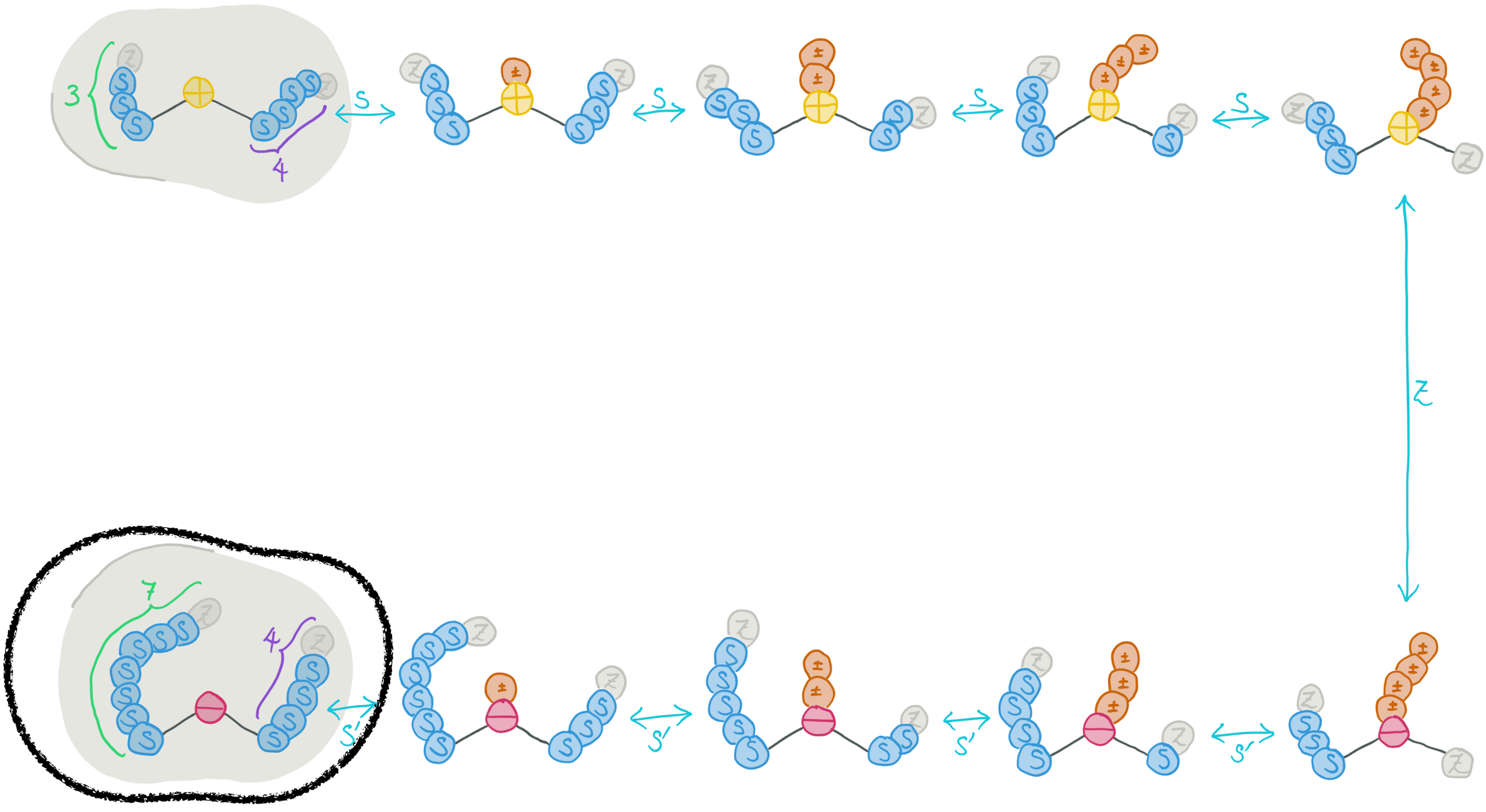

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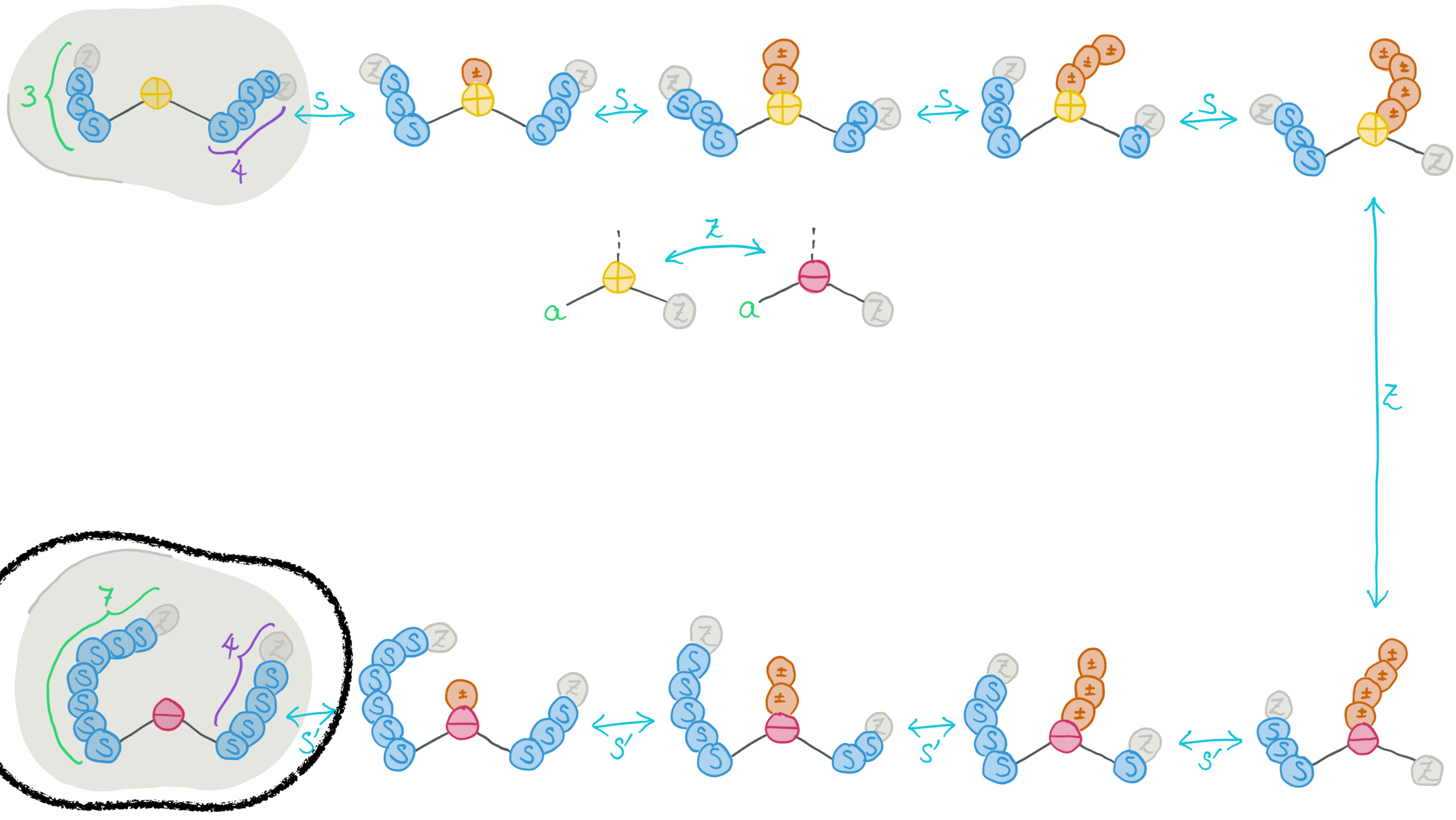

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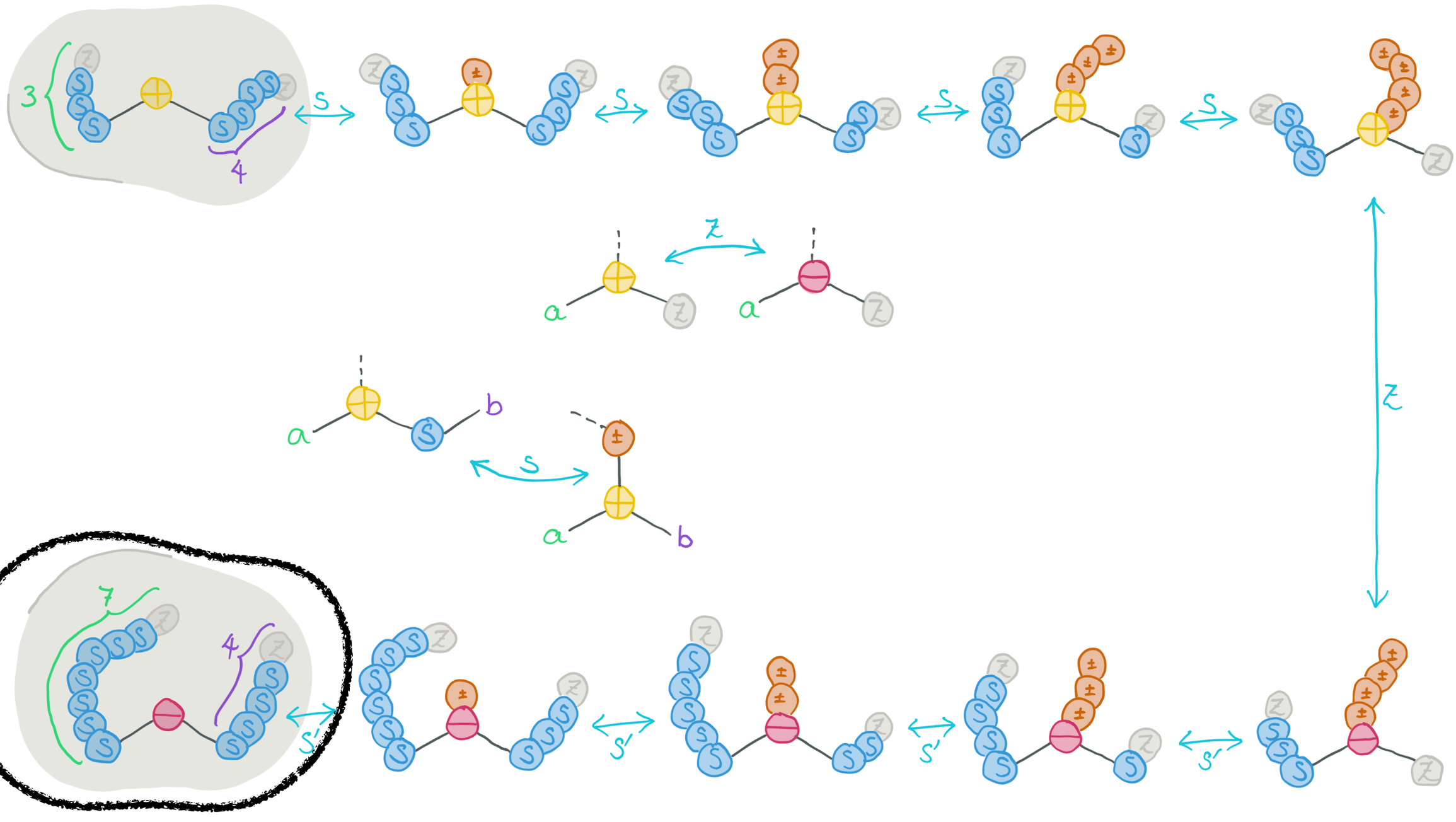

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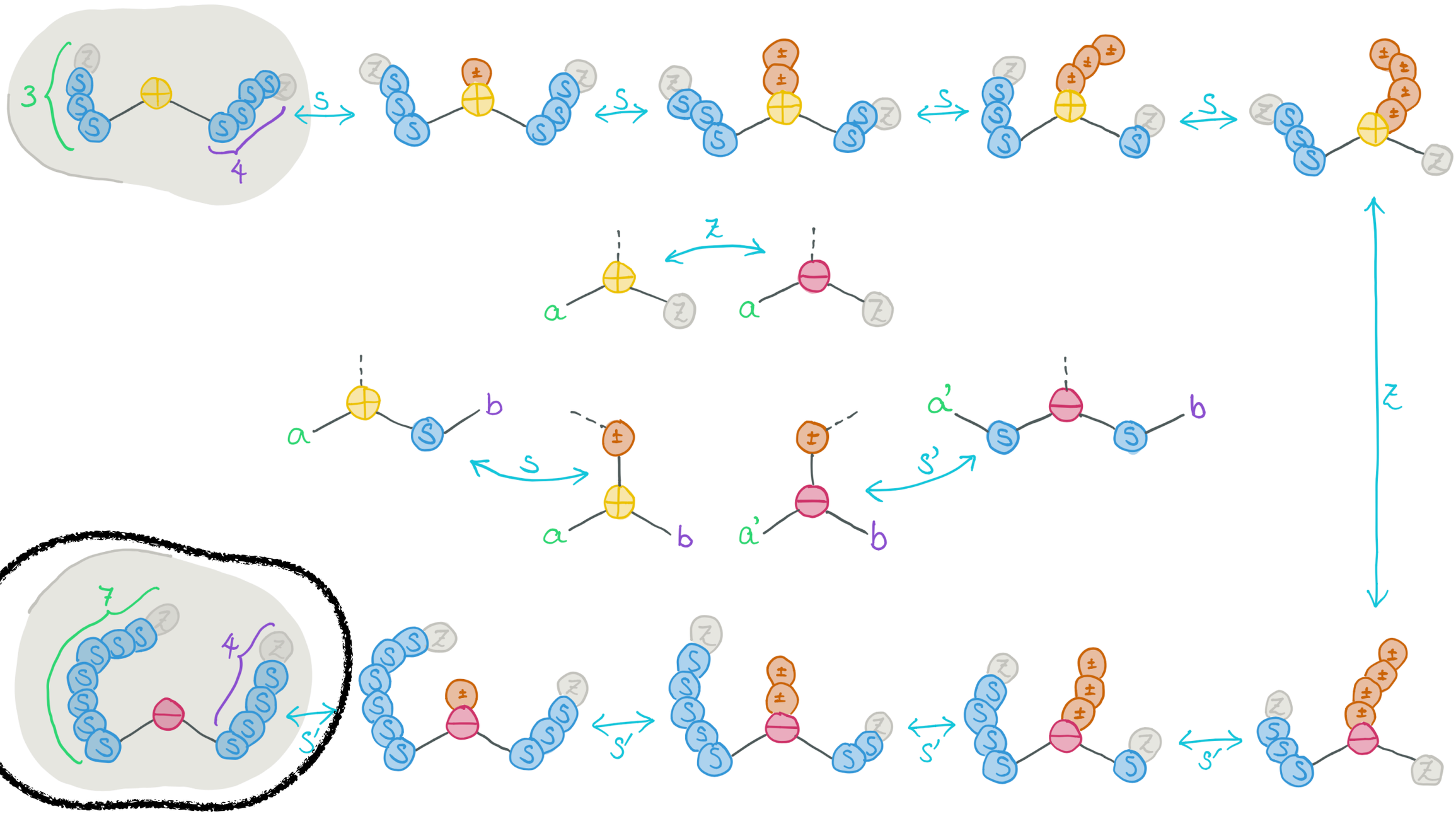

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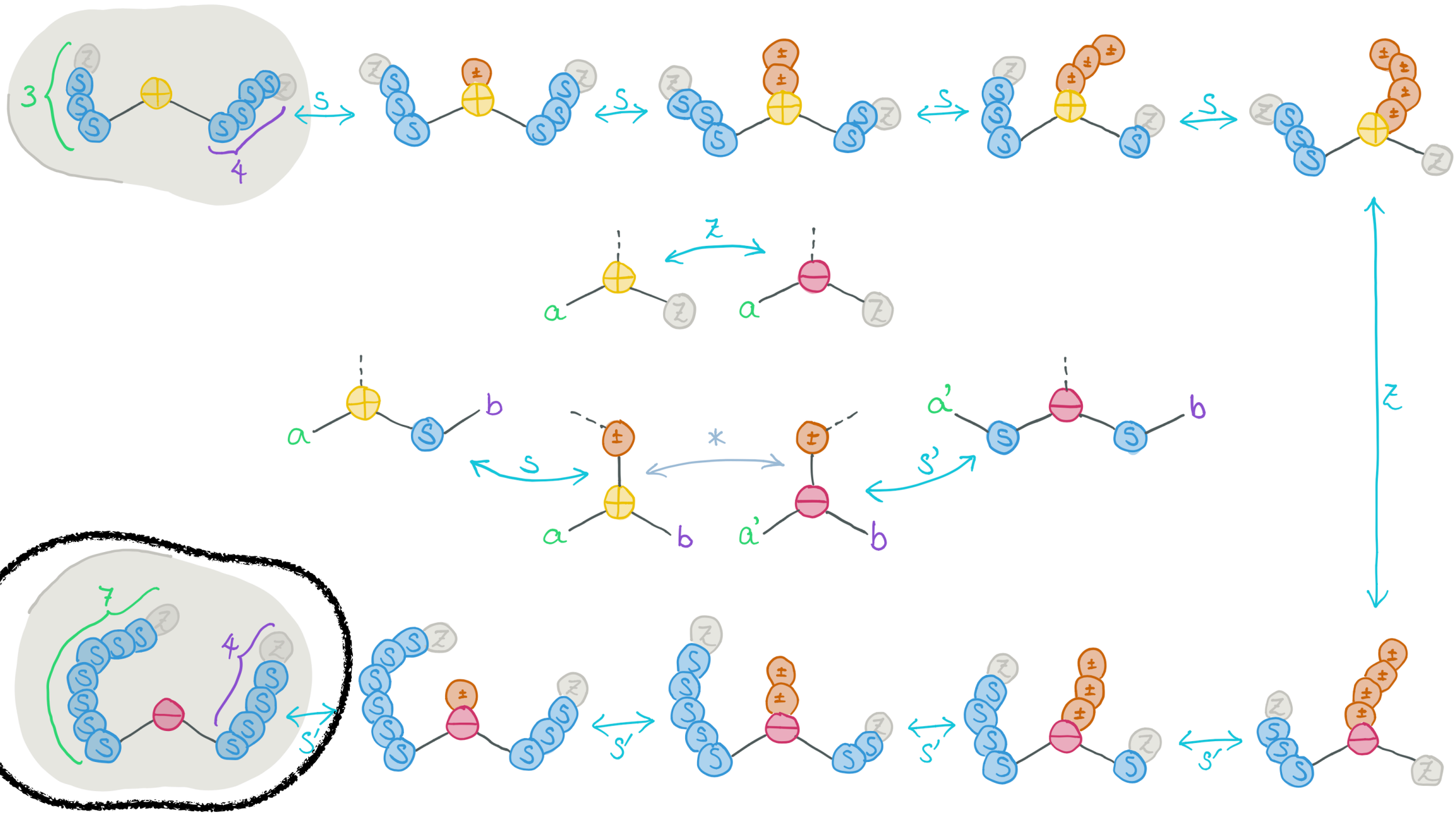

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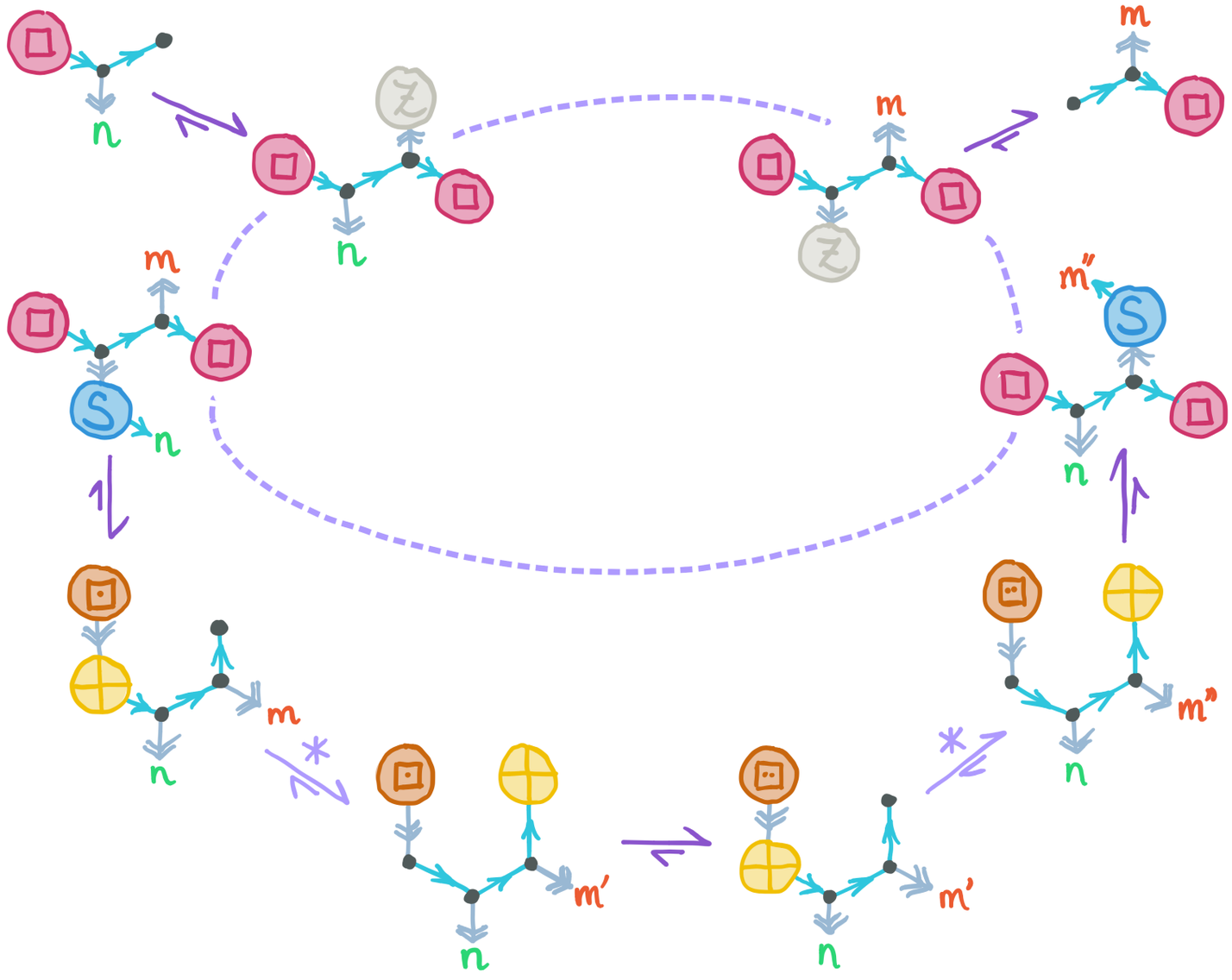

William Earley — 2020 — 'The א Calculus' — arXiv
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Alethe

```
% > 3 4 + x y
() 7 4 +
  x → 7
  y → 4
% < a b + 7 4
+ 3 4 ()
  a → 3
  b → 4
%
```

```
a  Z  +  a    Z;
a (S b) + (S c) (S b):
    a b + c b.
```

```
% > 7 ^2 z
() 49 ^2
  z → 49
% < c ^2 49
^2 7 ()
  c → 7
%
```

```
n ^2 n2:
  ! Go n     Z = Go Z n2.
    Go (S n) m = Go n (S k):
      m n + l n.
      l n + k n.
```

```
% > 2 9 `Pair` r
() 75 Pair
  r → 75
% < p q `Pair` 75
Pair 2 9 ()
  p → 2
  q → 9
%
```

```
a b `Pair` n:
  ! Go   n     Z   Z = Go Z   a    b.
    Go (S n)   Z   b = Go n (S b)  Z;
    Go (S n) (S a) b = Go n   a  (S b);
```

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Thank you!

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MICKLEM LAB

Engineering and Physical Sciences Research Council

Department of Applied Mathematics and Theoretical Physics (DAMTP)

William Earley — 2016–2021 — 'Modelling approaches to molecular computation' — EPSRC Project Reference 1781682
William Earley — 2020 — 'Engines of Parsimony: Part I; Limits on Computational Rates in Physical Systems' — arXiv
William Earley — 2020 — 'Engines of Parsimony: Part II; Performance Trade-offs for Communicating Reversible Computers' — arXiv
William Earley — 2020 — 'Engines of Parsimony: Part III; Performance Trade-offs for Reversible Computers Sharing Resources' — arXiv
William Earley — 2020 — 'The א Calculus' — arXiv