The ℵ-Calculus

a declarative model of reversible programming

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The ℵ-Calculus

- declarative
- reversible TRS semantics, without history
- minimalistic definition

$$
\begin{aligned}
&(\text{PATTERN TERM}) && \pi ::= \text{SYM} \mid \text{VAR} \mid (\ \pi^{*}\ ) \\
&(\text{RULE}) && \rho ::= \pi^{*} = \pi^{*} \\
&(\text{DEFINITION}) && \delta ::= \rho : \rho.^{*} \mid \mathbf{!}\ \pi^{*};
\end{aligned}
$$

Addition

$$\textbf{!}\ +\ a\ b\ ();\quad \textbf{!}\ ()\ c\ b\ +;$$

$$+\ a\ \mathrm{Z}\ () = ()\ a\ \mathrm{Z}\ +; \qquad (\text{ADD–BASE})$$

$$+\ a\ (\mathrm{S}b)\ () = ()\ (\mathrm{S}c)\ (\mathrm{S}b)\ +: \qquad (\text{ADD–STEP})$$

$$+\ a\ b\ () = ()\ c\ b\ +. \qquad (\text{ADD–STEP–SUB})$$

(PATTERN TERM)	$\pi ::= \text{SYM} \mid \text{VAR} \mid (\ \pi^*\ )$
(RULE)	$\rho ::= \pi^* = \pi^*$
(DEFINITION)	$\delta ::= \rho : \rho^*. \mid \textbf{!}\ \pi^*;$

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$$\mathbf{!}\quad +\ 3\ 2\ () \qquad (\textsc{add–step})$$

Addition

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$$\mathbf{!}\quad +\ 3\ 2\ () \leftrightsquigarrow \underline{\{a \mapsto 3, b \mapsto 1\}} \qquad \text{(ADD–STEP)}$$

$$\text{(ADD–STEP–SUB)}$$

$$\overline{+\ 3\ 1\ ()} \qquad \text{(ADD–STEP)}$$

Addition

$$
\begin{array}{ll}
\textbf{!}\ + a\ b\ ();\quad \textbf{!}\ ()\ c\ b\ +; & \\
\quad + a\ \mathrm{Z}\ () = ()\ a\ \mathrm{Z}\ +; & (\textsc{add–base}) \\
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\end{array}
$$

$$
\begin{array}{ll}
\textbf{!}\quad + 3\ 2\ () \leftrightsquigarrow \underline{\{a \mapsto 3, b \mapsto 1\}} & (\textsc{add–step}) \\
 & (\textsc{add–step–sub}) \\
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\overline{+ 3\ \mathrm{Z}\ ()} &
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$$(\textsc{add–step–sub})$$

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$$\overline{+\ 3\ \mathrm{Z}\ ()}\ \leftrightsquigarrow\ \{a \mapsto 3\}\ \leftrightsquigarrow\ ()\ 3\ \mathrm{Z}\ + \qquad (\textsc{add–base})$$

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$$\overline{+\ 3\ 1\ ()} \leftrightsquigarrow \underline{\{a \mapsto 3, b \mapsto 0\}} \qquad \underline{\{c \mapsto 3, b \mapsto 0\}} \leftrightsquigarrow \overline{()\ 4\ 1\ +} \qquad (\textsc{add–step})$$

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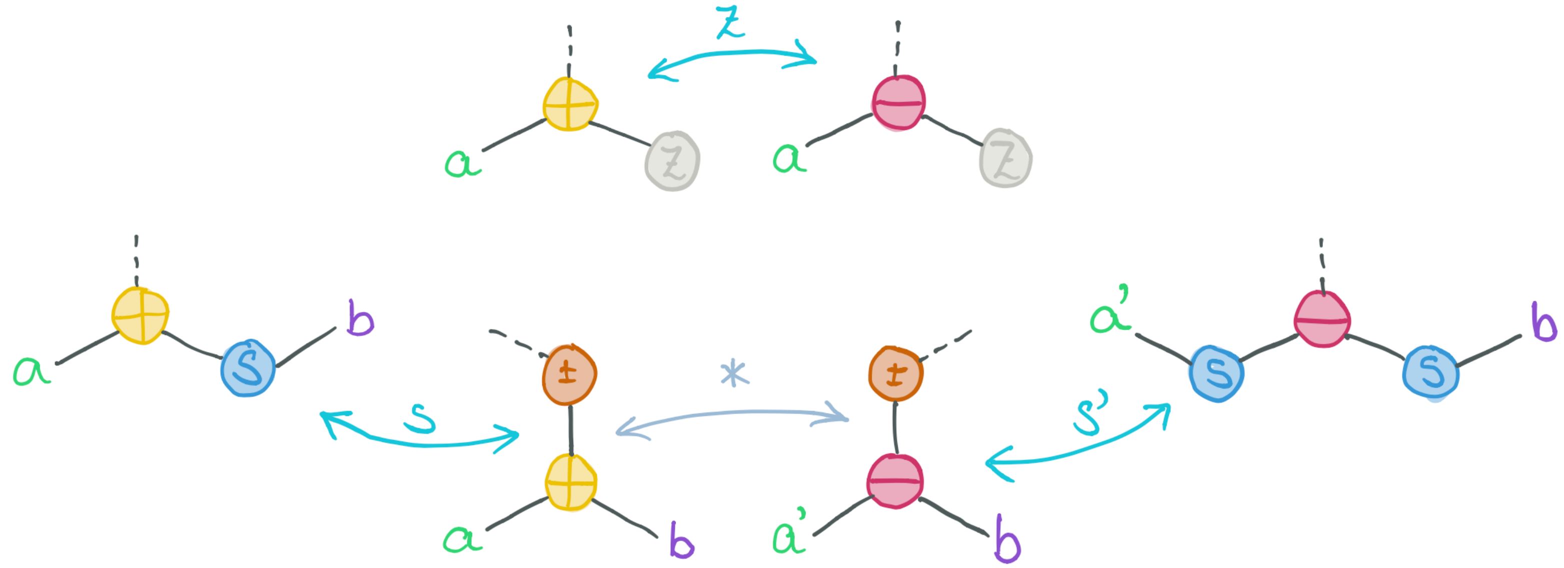
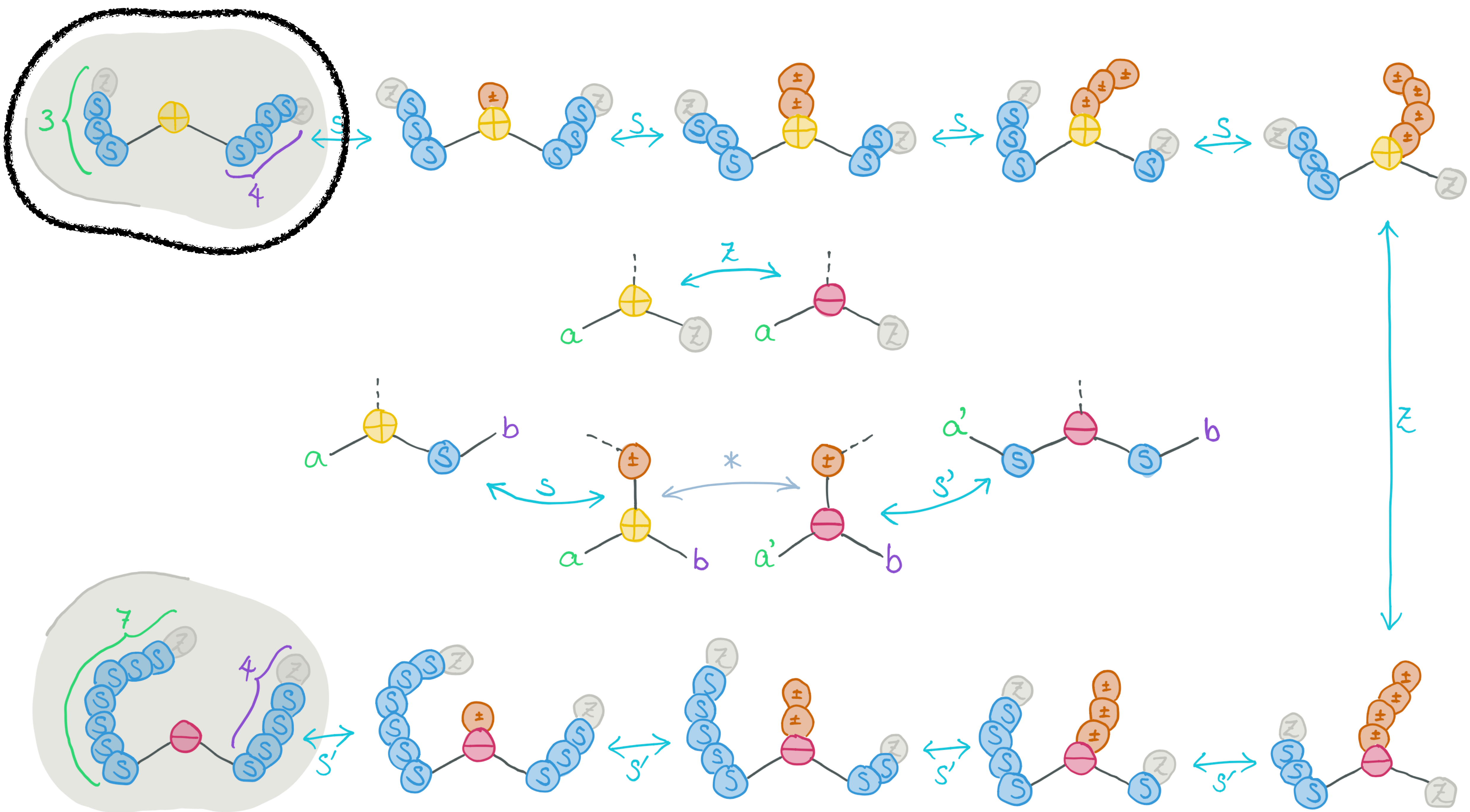


Addition



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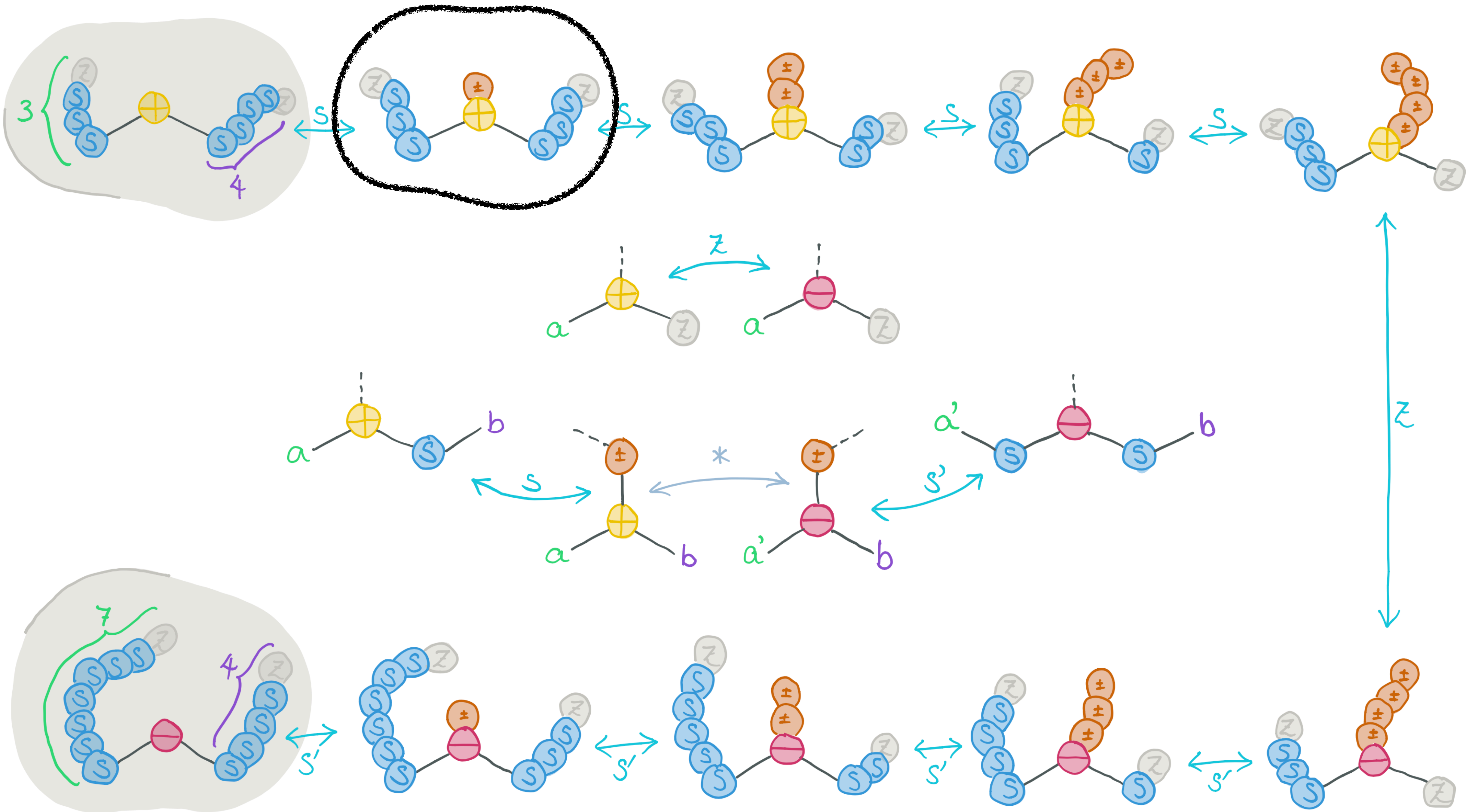

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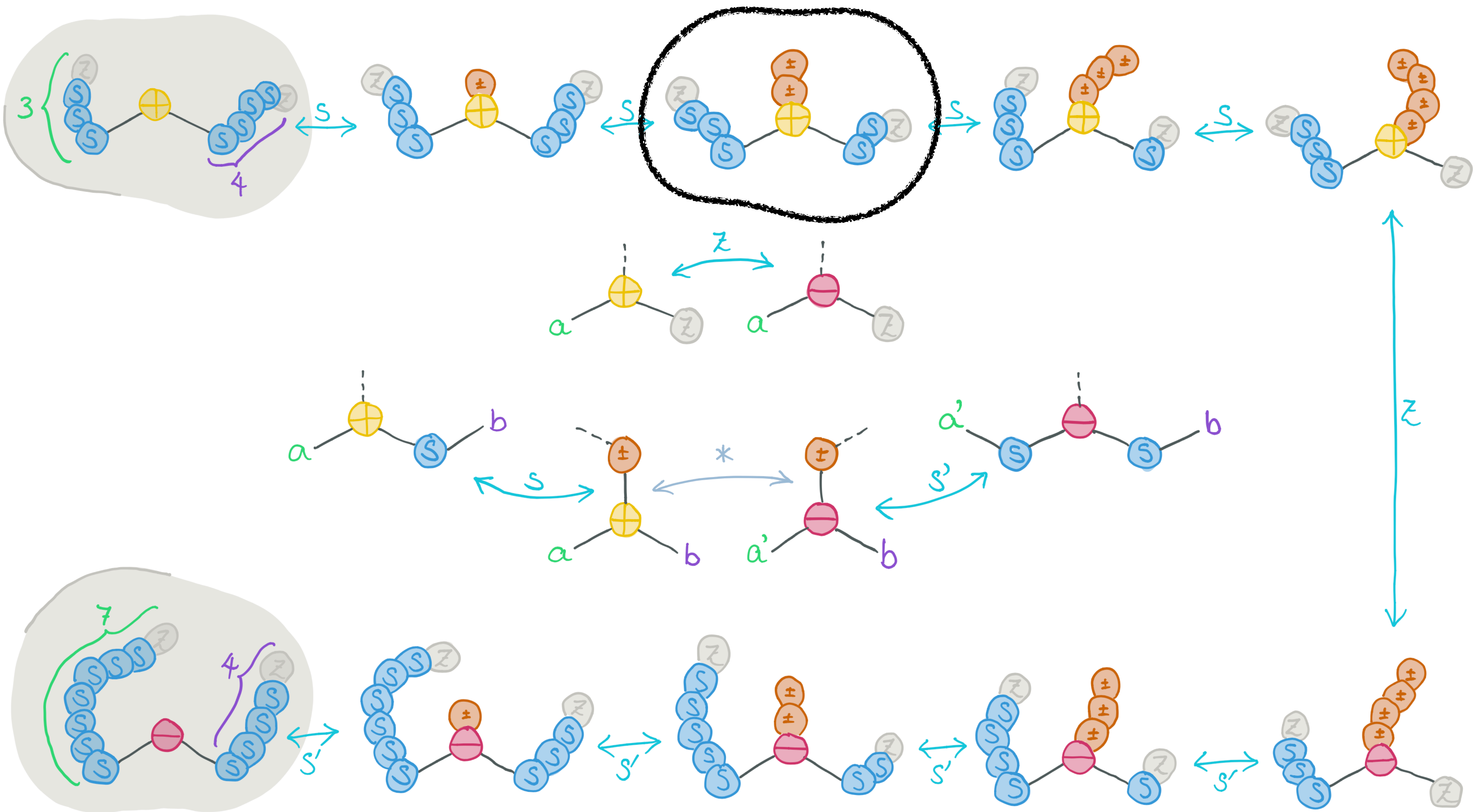

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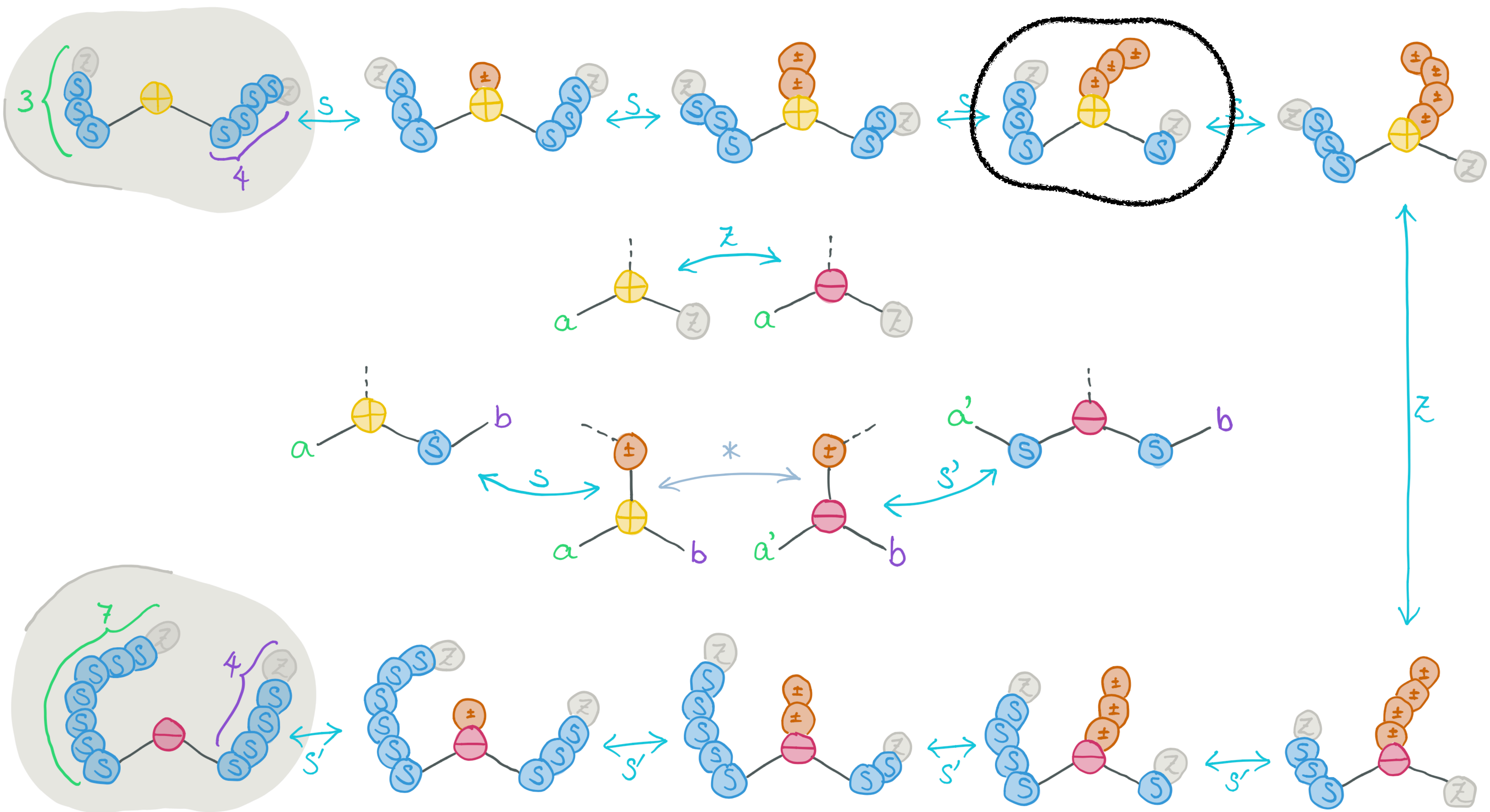

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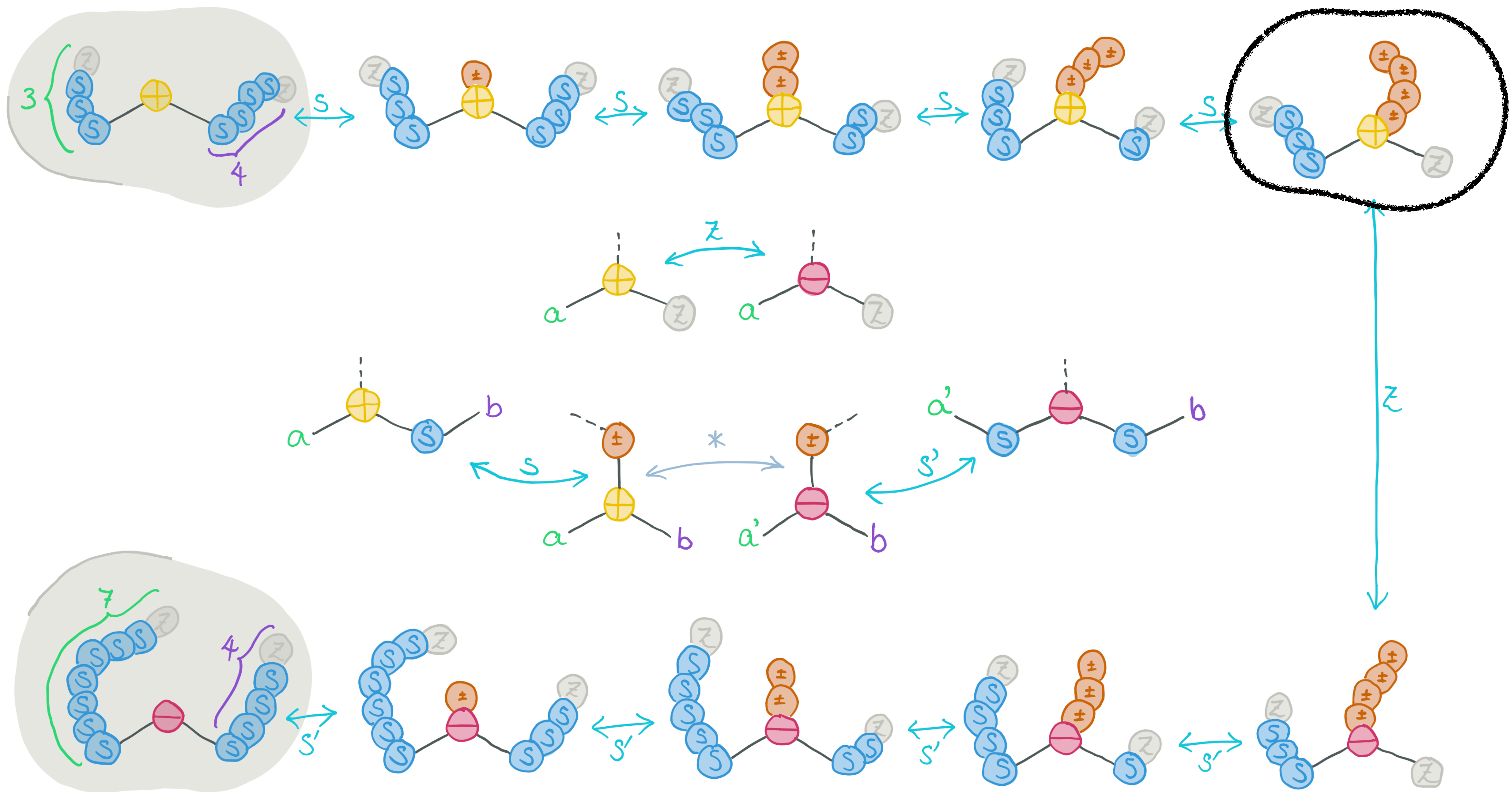

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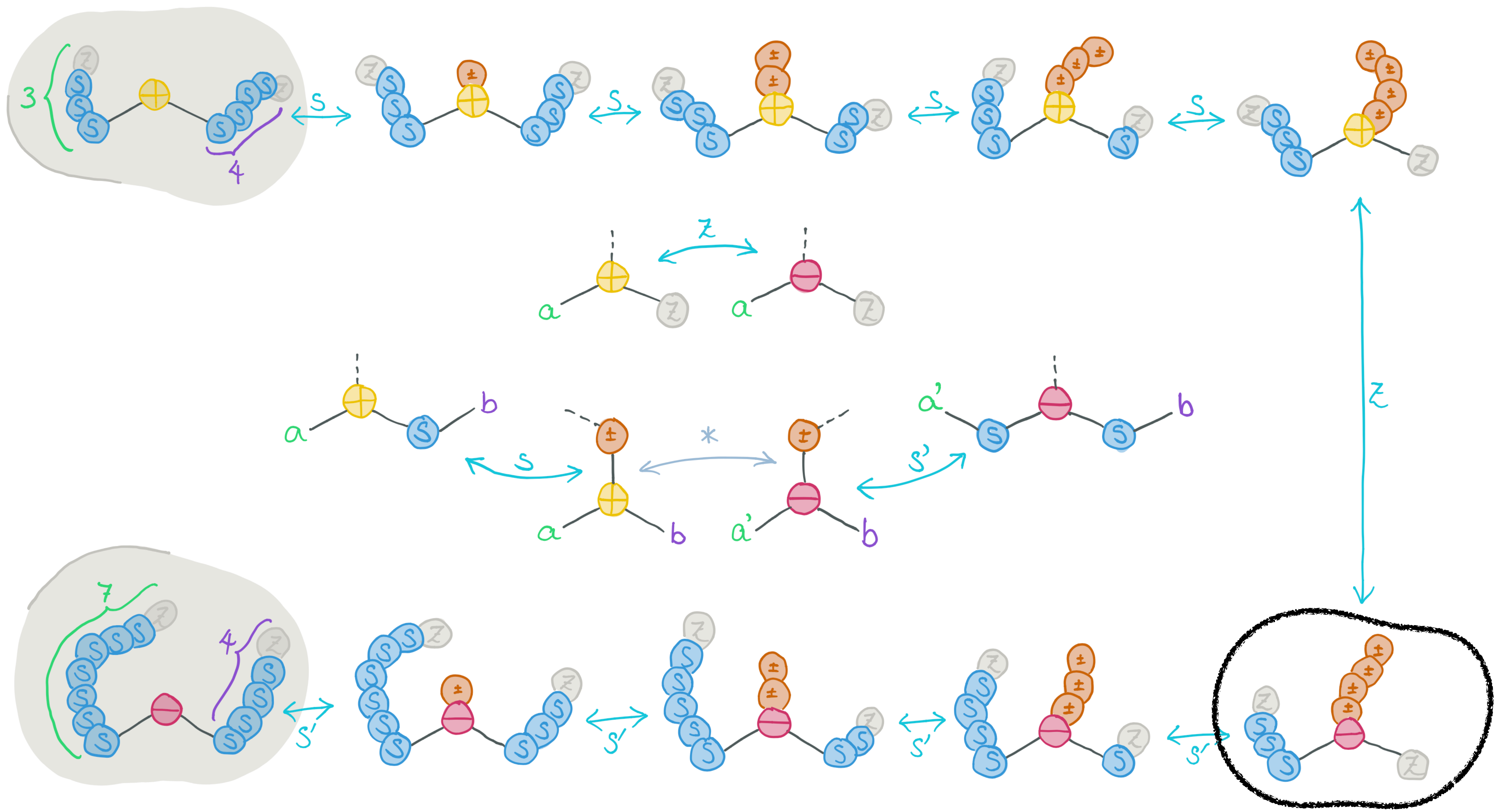

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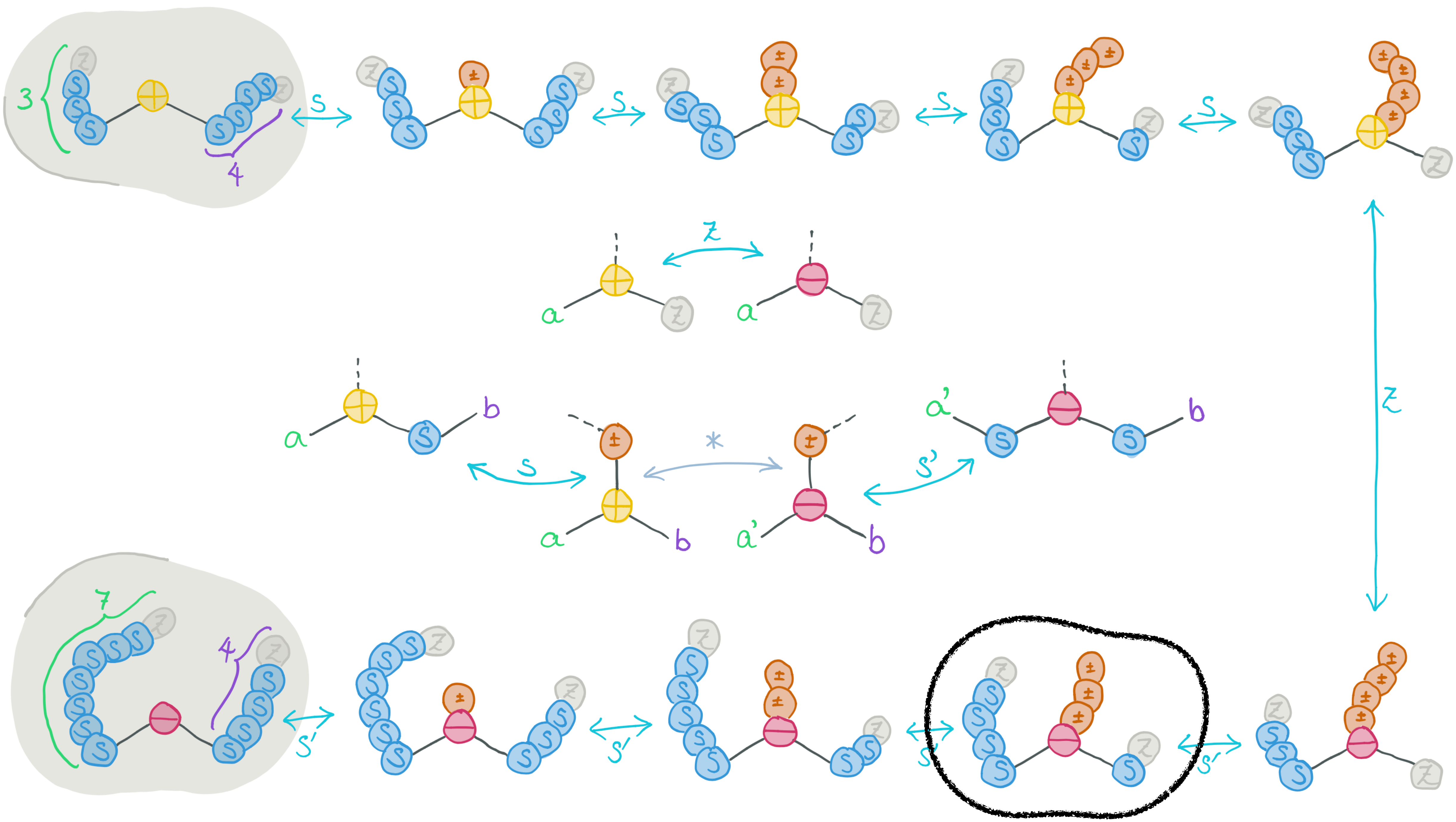

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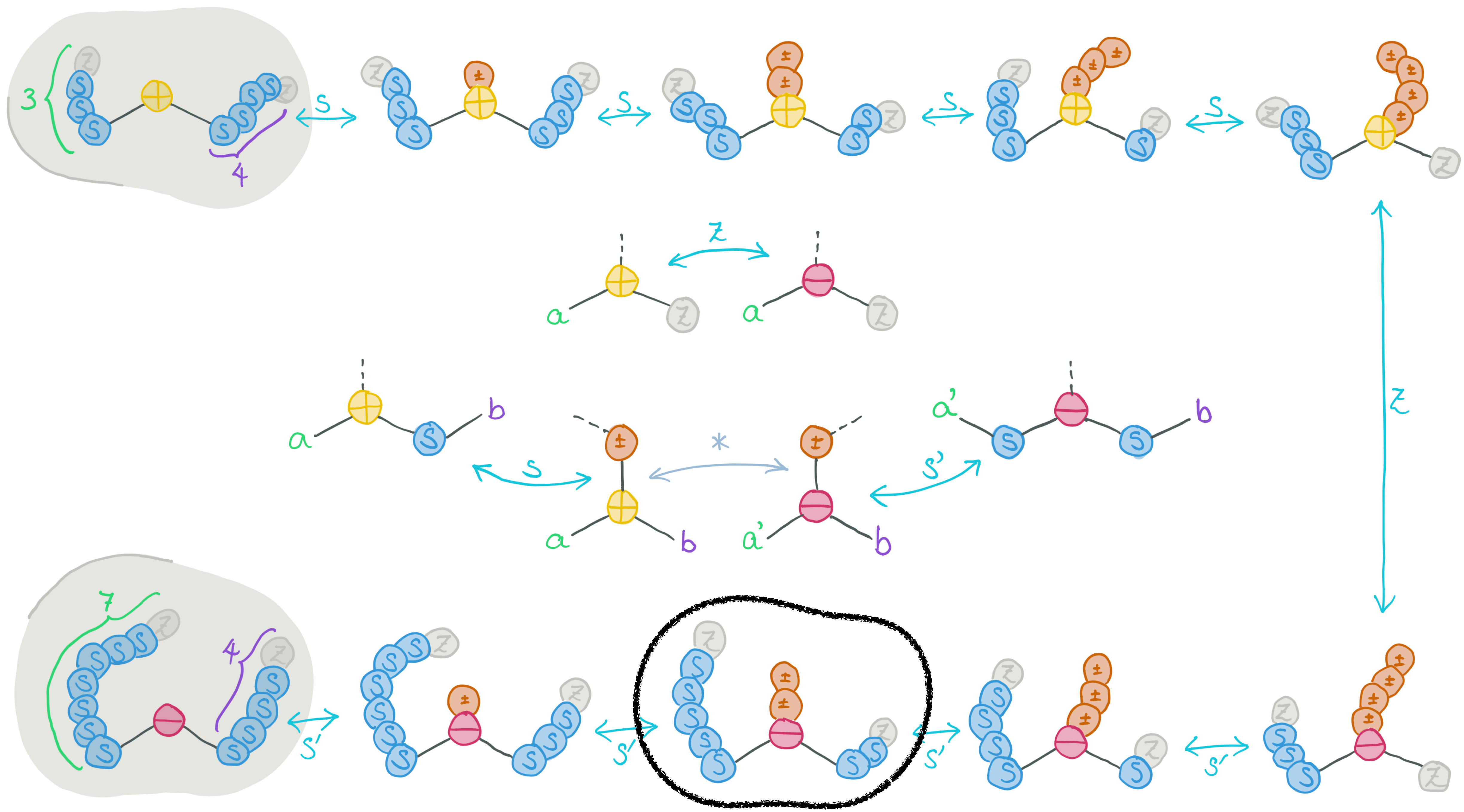

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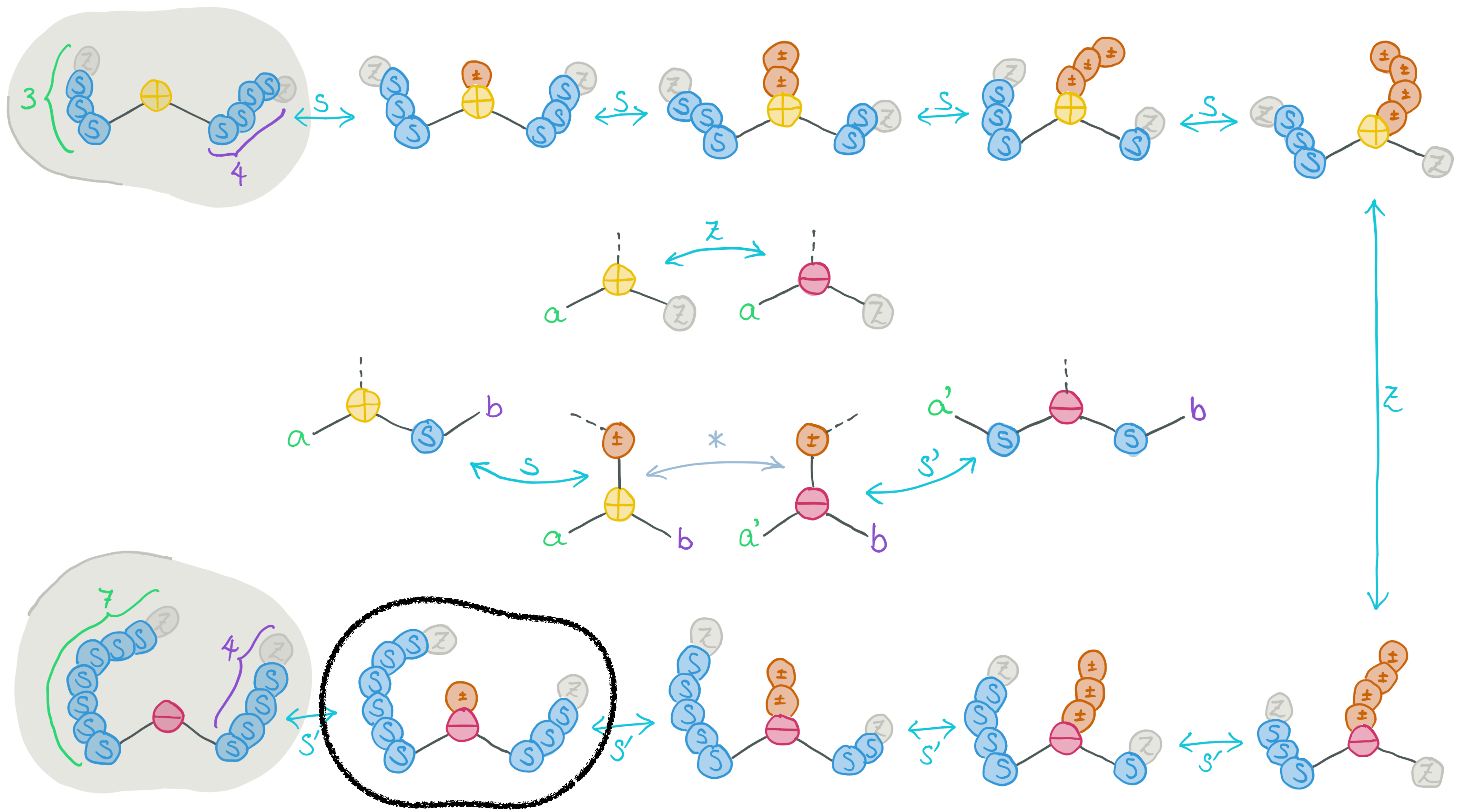

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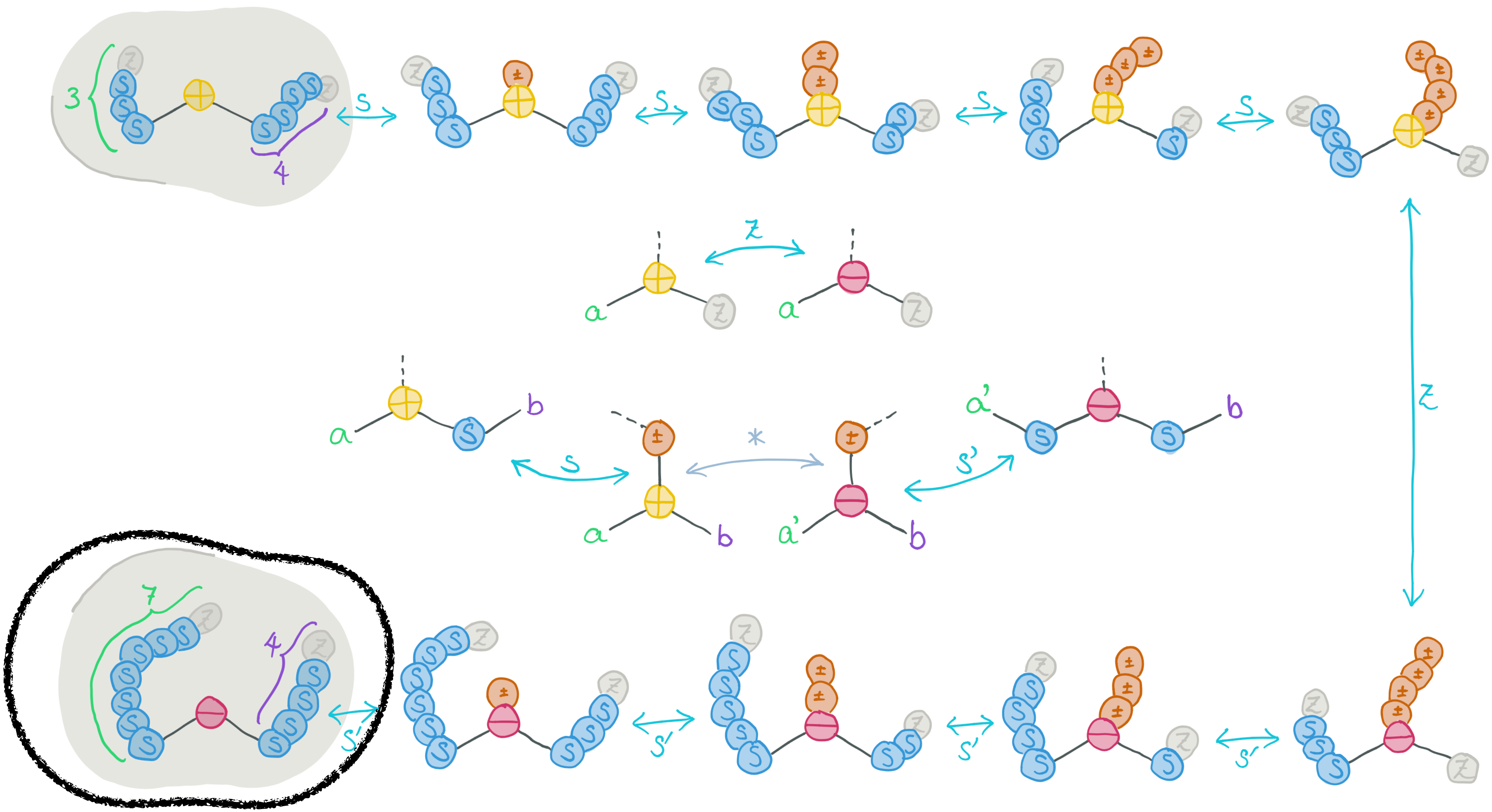

Squaring

$$m^2 = \sum_{k=0}^{m-1} (k + k + 1)$$

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$\textrm{S}\textsc{q}\ s\ (\textrm{S}k)\ \textrm{S}\textsc{q} = \textrm{S}\textsc{q}\ (\textrm{S}s'')\ k\ \textrm{S}\textsc{q}:$ (SQ–STEP)

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$\textrm{S}\textsc{q}\ n\ \textrm{Z}\ \textrm{S}\textsc{q} = ()\ n\ \textrm{S}\textsc{q};$ (SQ–END)

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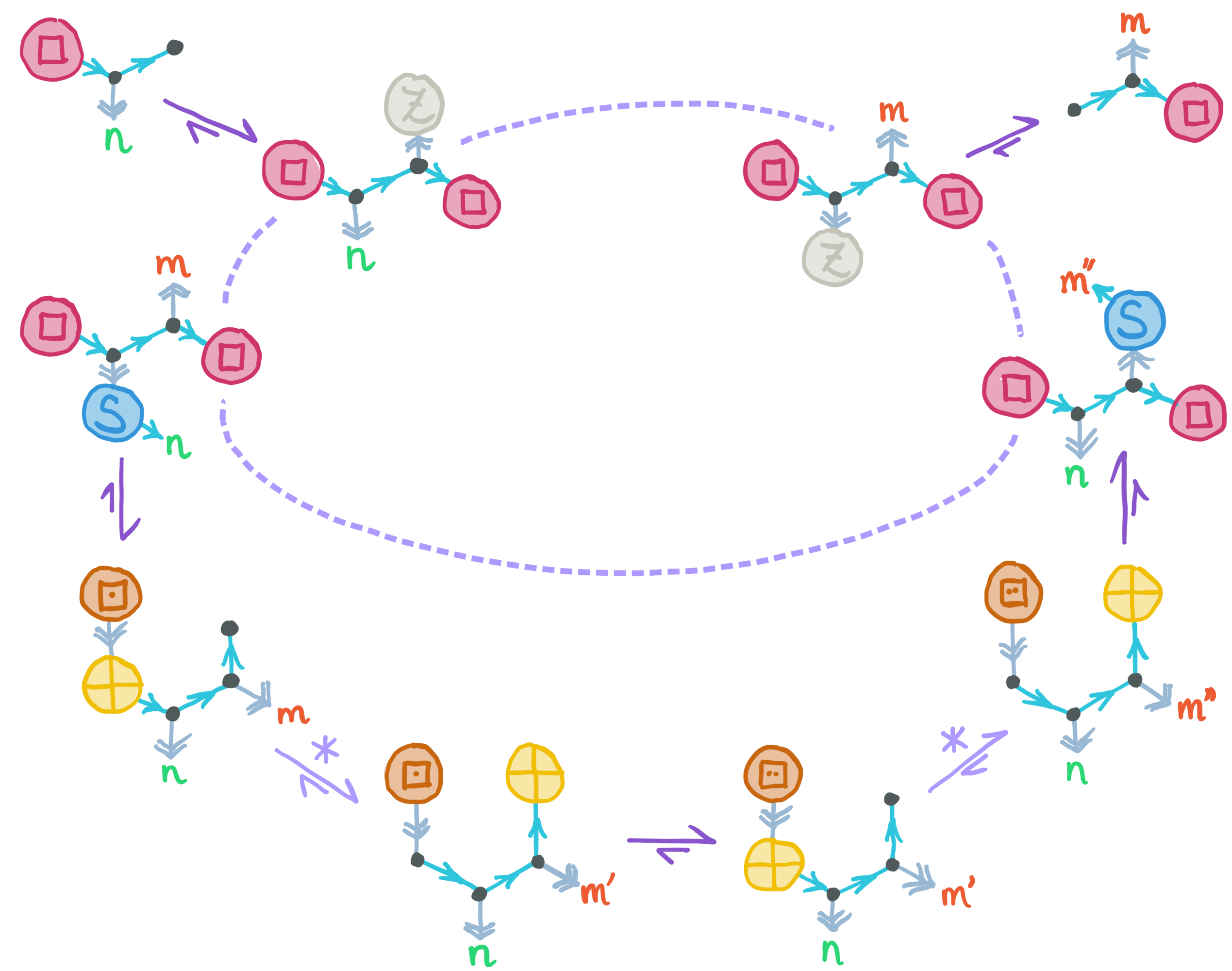

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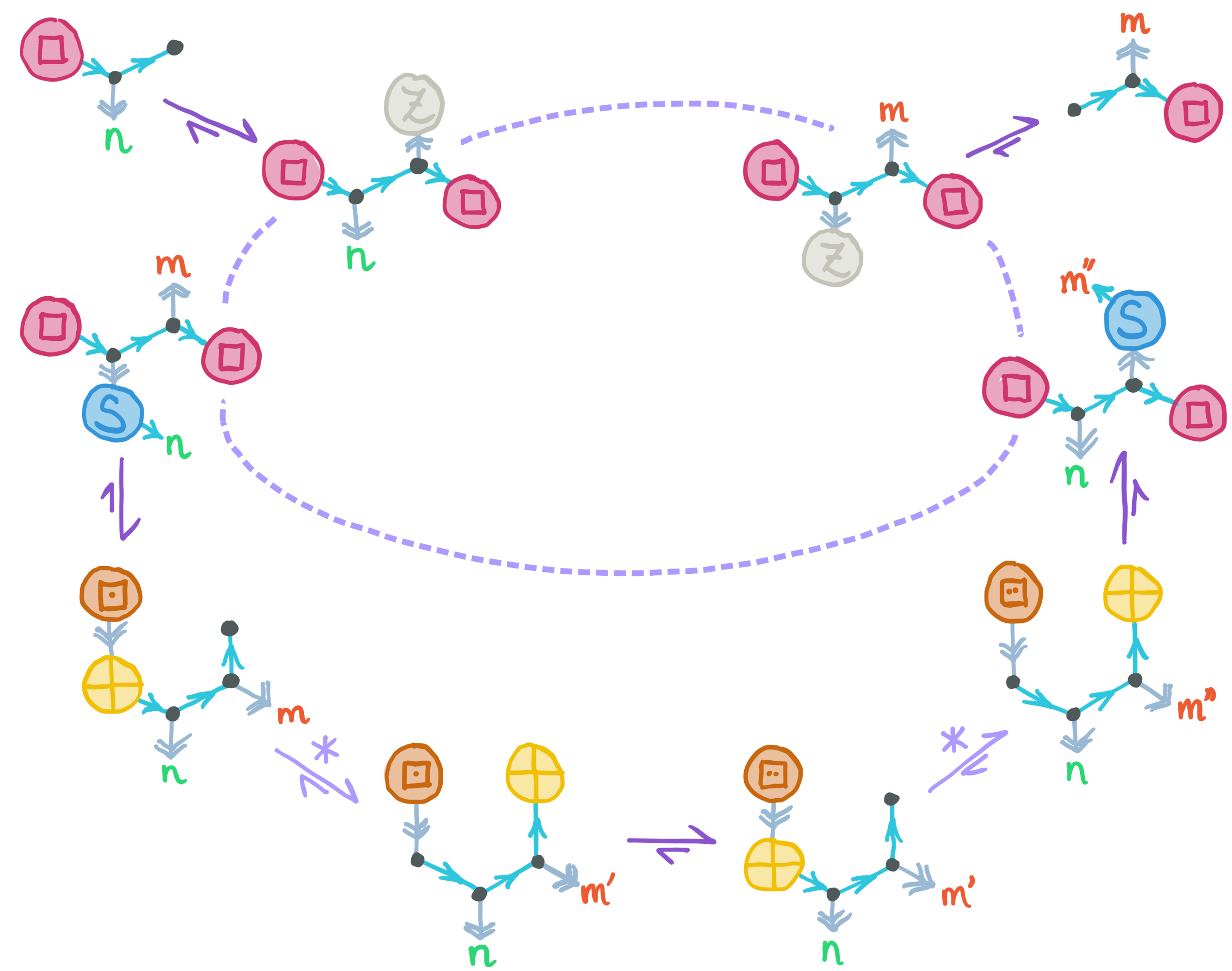

$\textbf{!}\quad \textsc{Sq}\ 3\ ()$

Squaring

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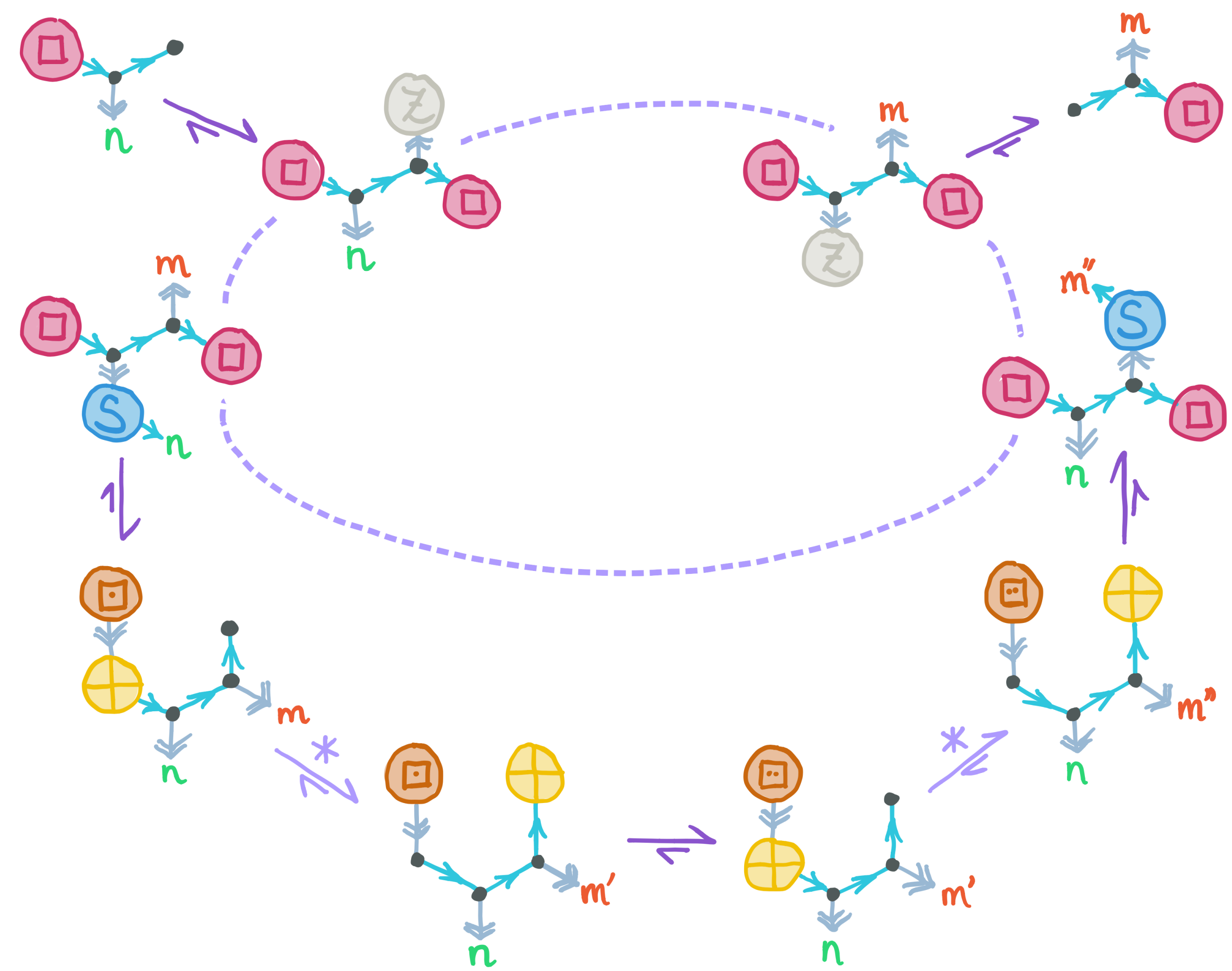

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$\mathbf{!}\quad \textsc{Sq}\ 3\ () = \textsc{Sq}\ \mathrm{Z}\ 3\ \textsc{Sq}$

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$$
\begin{array}{ll}
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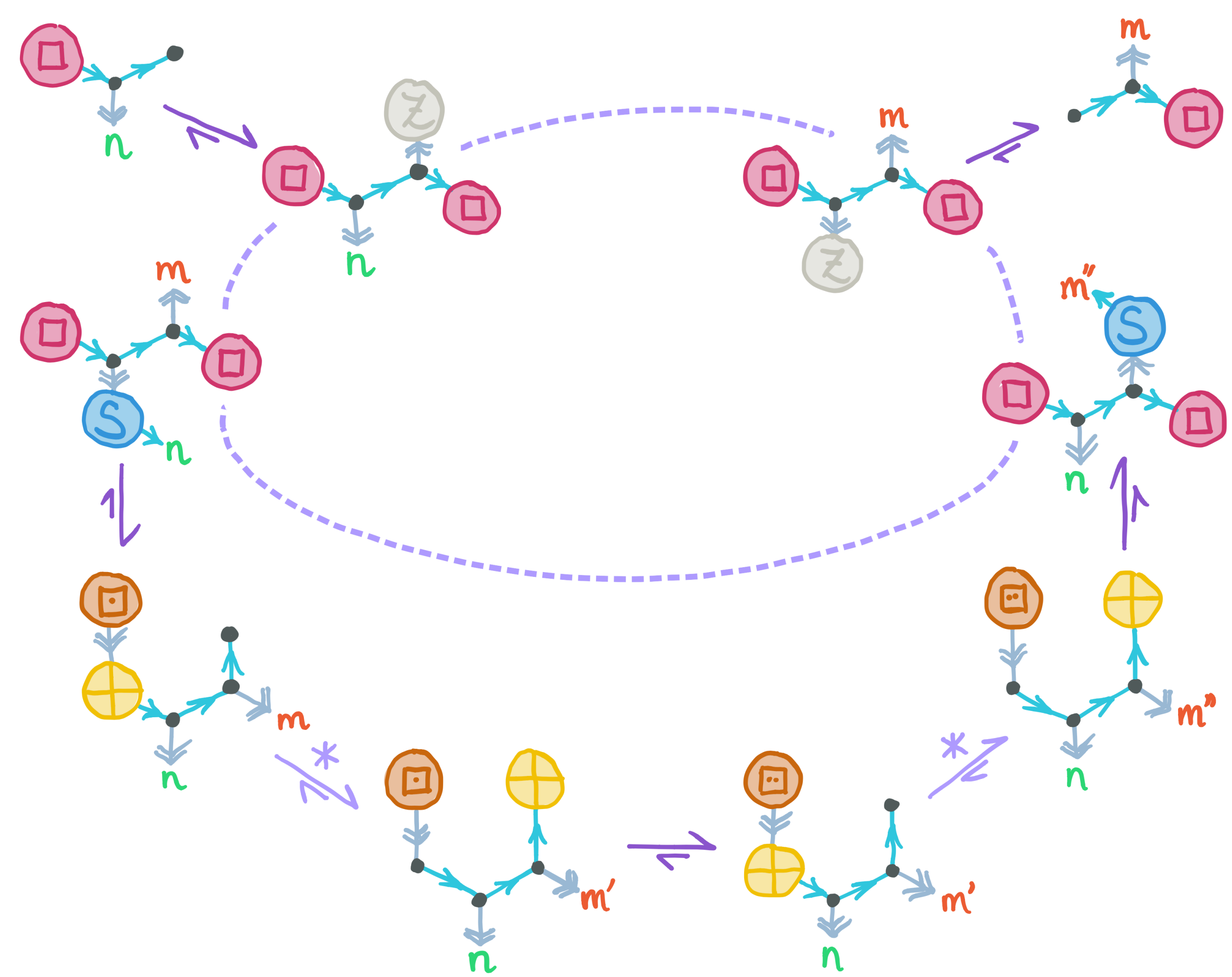
\end{array}
$$



$$\mathbf{!}\quad \textsc{Sq}\ 3\ () = \textsc{Sq}\ \mathrm{Z}\ 3\ \textsc{Sq} = \textsc{Sq}\ 5\ 2\ \textsc{Sq}$$

Squaring

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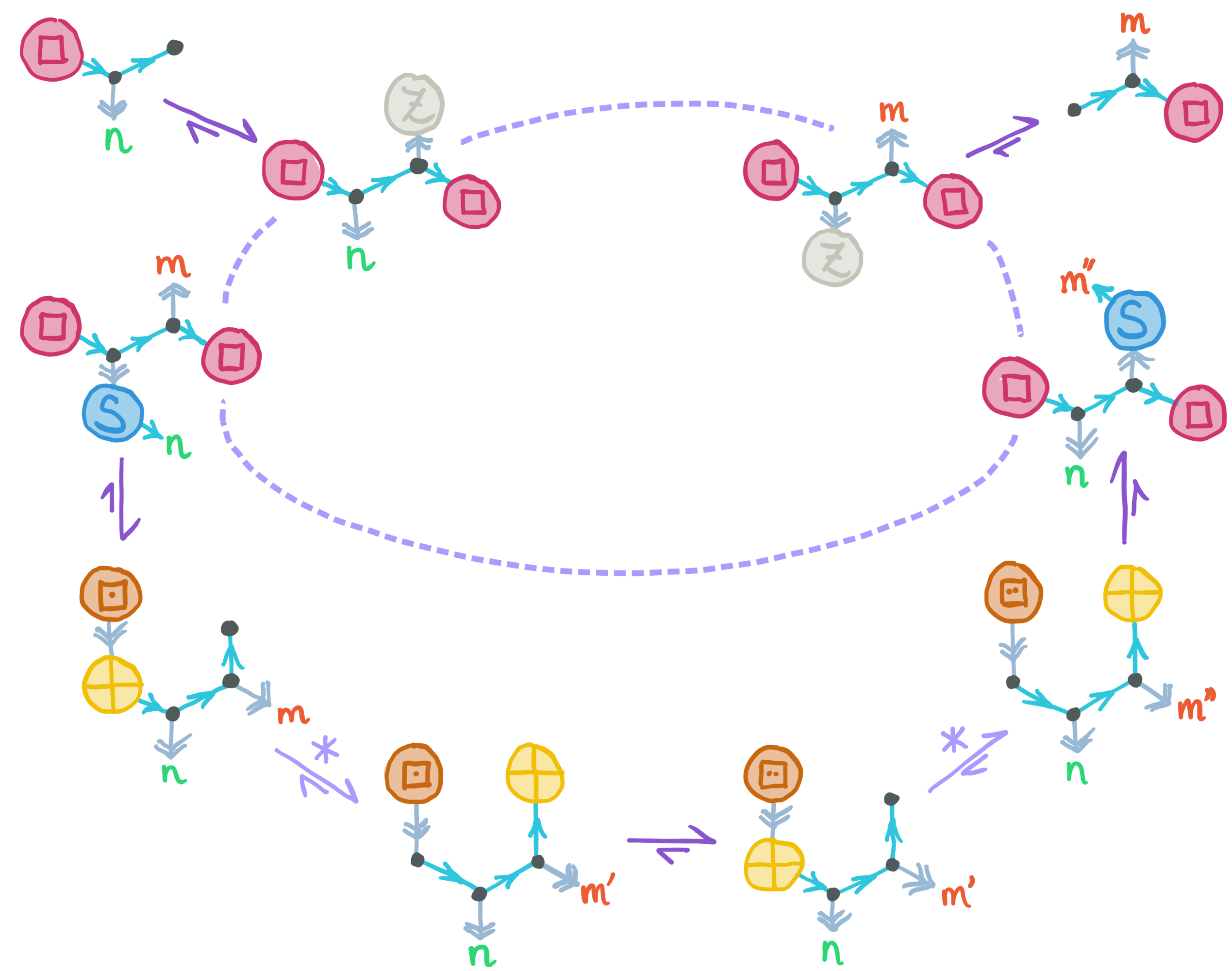

$! \ \textsc{Sq}\ m\ ();\quad !\ ()\ n\ \textsc{Sq};$

$\textsc{Sq}\ m\ () = \textsc{Sq}\ \mathrm{Z}\ m\ \textsc{Sq};$	(SQ–BEGIN)
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$\textsc{Sq}\ n\ \mathrm{Z}\ \textsc{Sq} = ()\ n\ \textsc{Sq};$	(SQ–END)

$! \quad \textsc{Sq}\ 3\ () = \textsc{Sq}\ \mathrm{Z}\ 3\ \textsc{Sq} = \textsc{Sq}\ 5\ 2\ \textsc{Sq} = \textsc{Sq}\ 8\ 1\ \textsc{Sq}$

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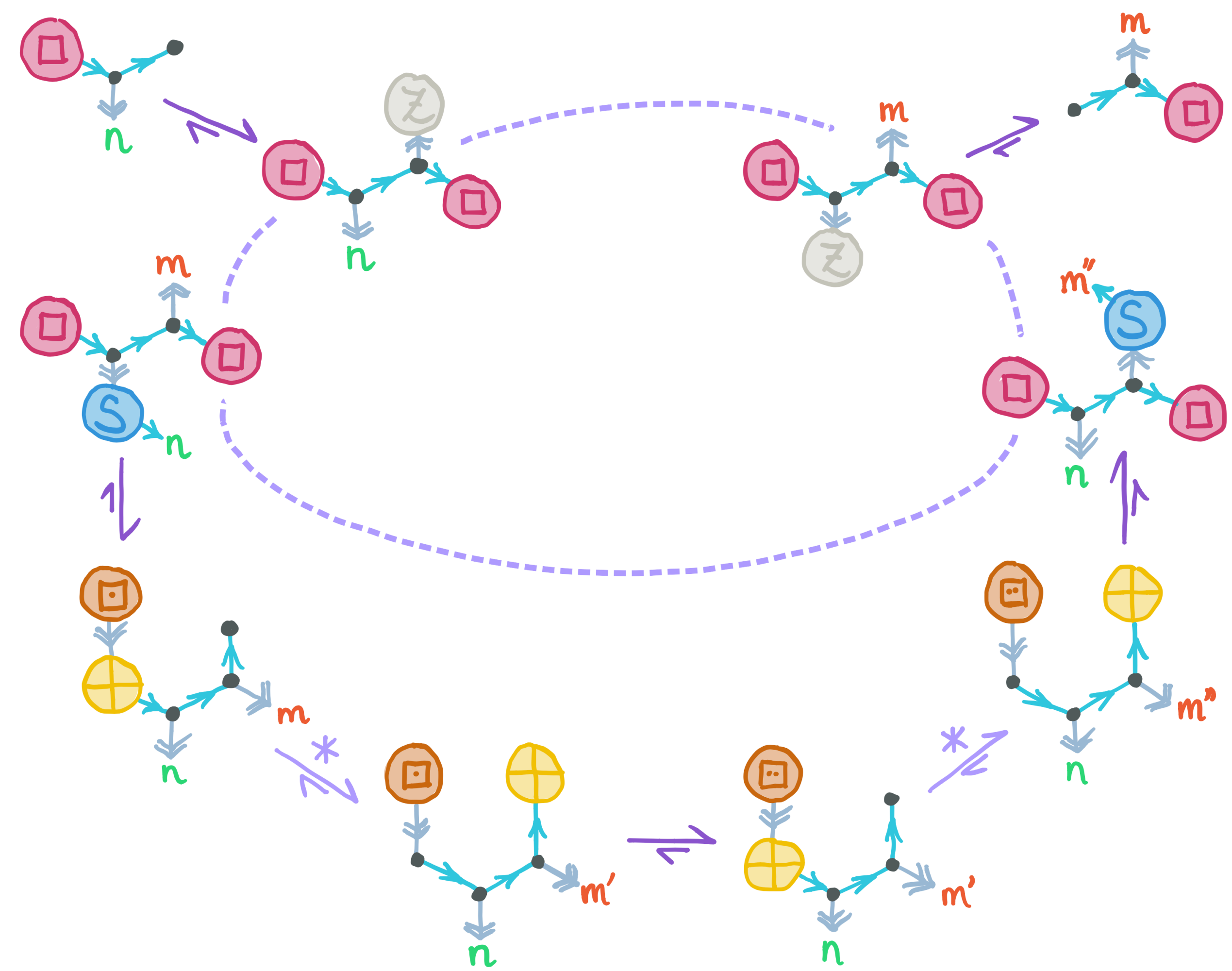

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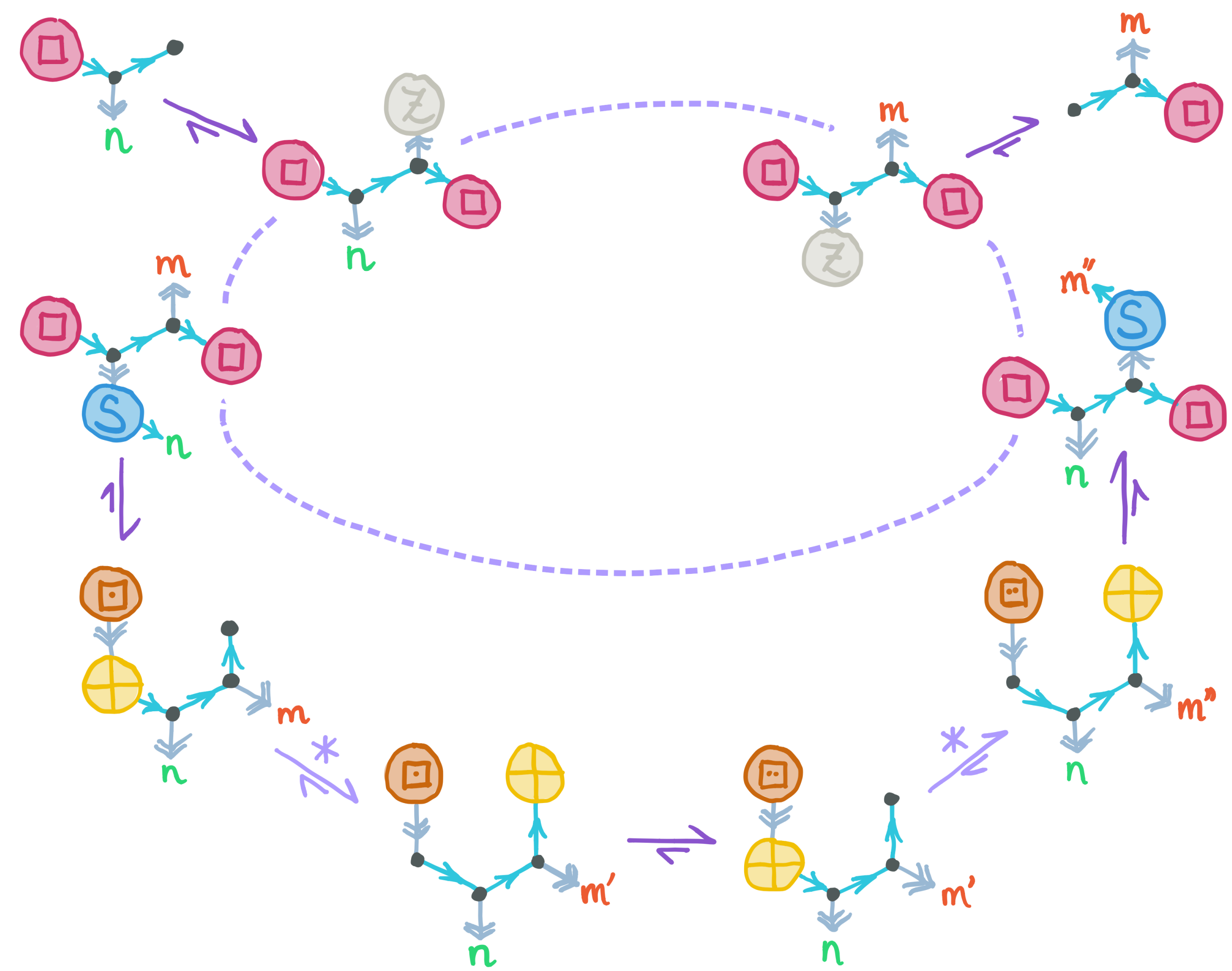

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$\mathbf{!}$ Sq m (); $\mathbf{!}$ () n Sq;

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Sq s (Sk) Sq = Sq (Ss'') k Sq:	(sq–step)
+ s k () = () s' k +.	-- $s' \leftarrow s + k$
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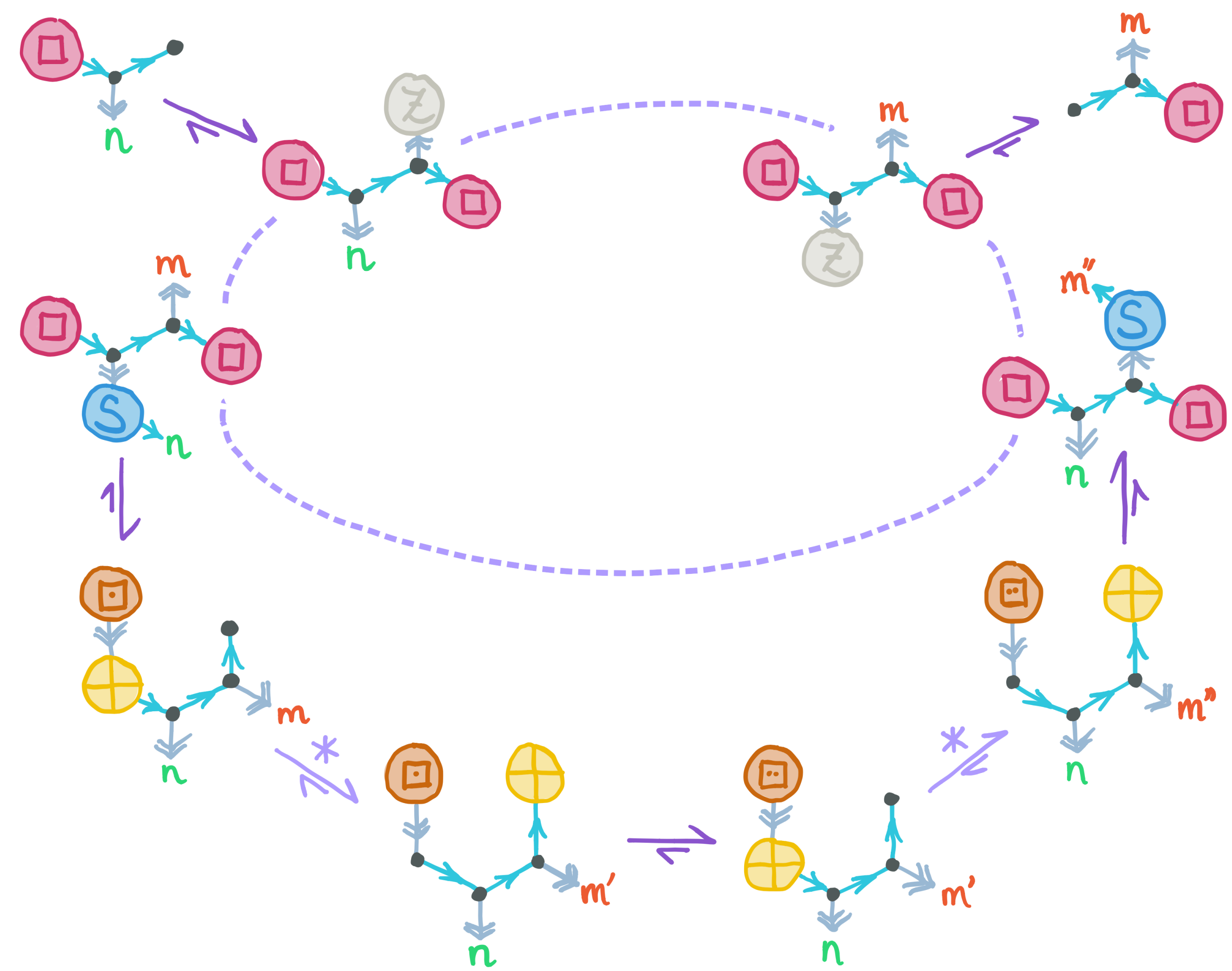
Sq n Z Sq = () n Sq;	(sq–end)



$\mathbf{!}$ () 10 Sq

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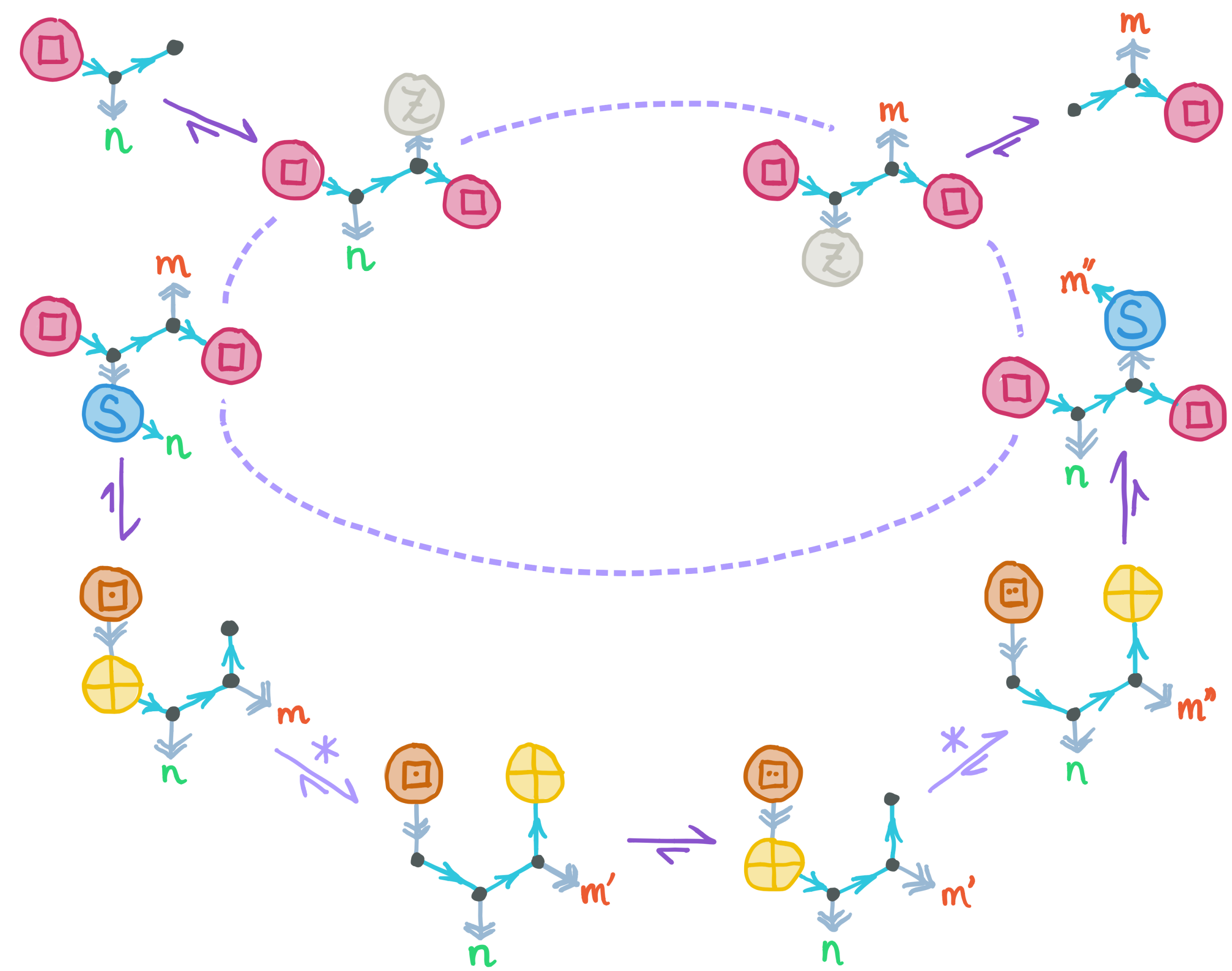
$\mathrm{SQ}\ n\ \mathrm{Z}\ \mathrm{SQ} = ()\ n\ \mathrm{SQ};$	(SQ–END)



$! \quad ()\ 10\ \mathrm{SQ} = \mathrm{SQ}\ 10\ \mathrm{Z}\ \mathrm{SQ}$

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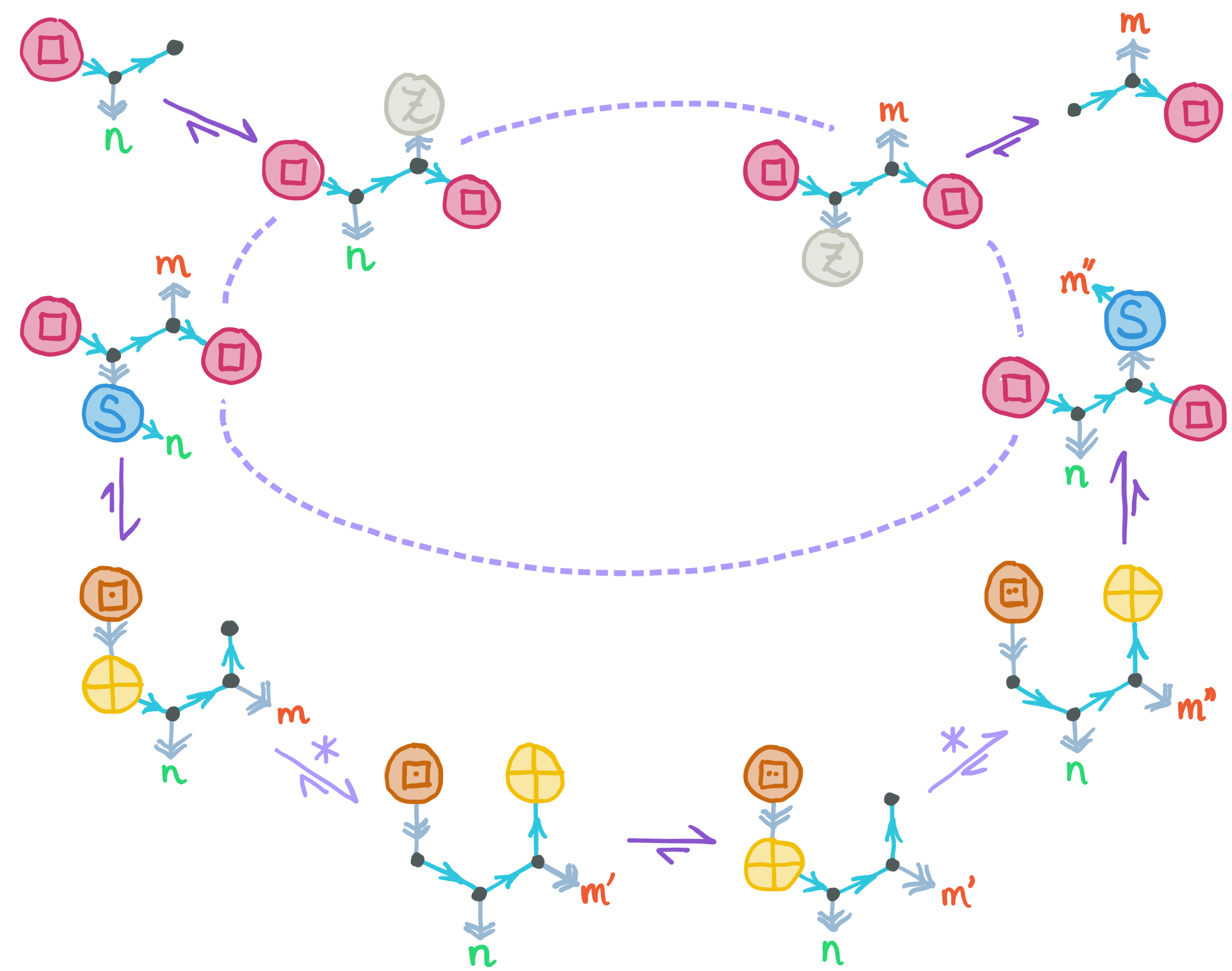

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\end{array}
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Squaring

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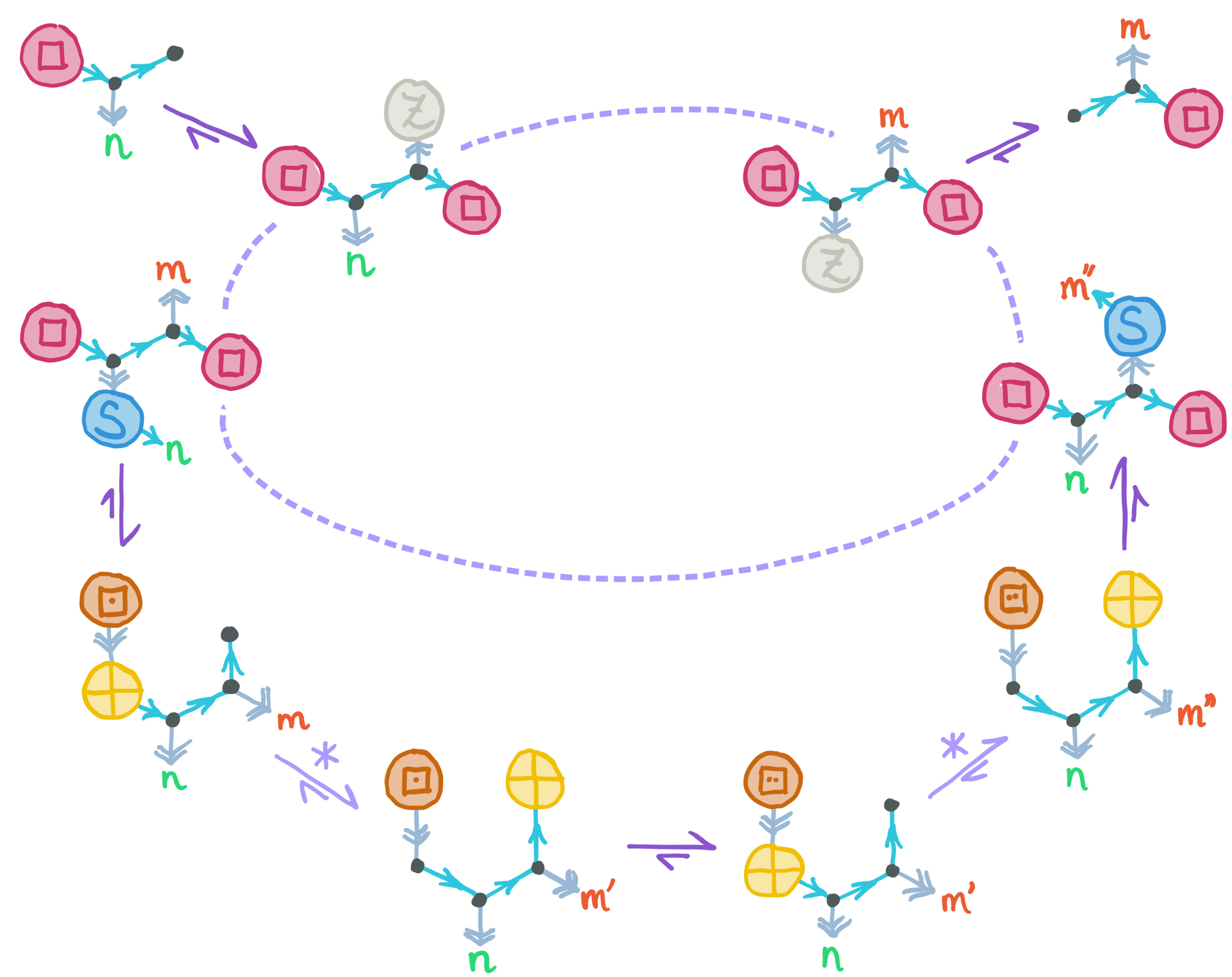

$\mathbf{!}$ Sq m (); $\mathbf{!}$ () n Sq;

Sq m () = Sq Z m Sq; (sq–begin)

Sq s (Sk) Sq = Sq (Ss'') k Sq: (sq–step)

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Sq n Z Sq = () n Sq; (sq–end)

$\mathbf{!}$ () 10 Sq = Sq 10 Z Sq = Sq 9 1 Sq = Sq 6 2 Sq

Squaring

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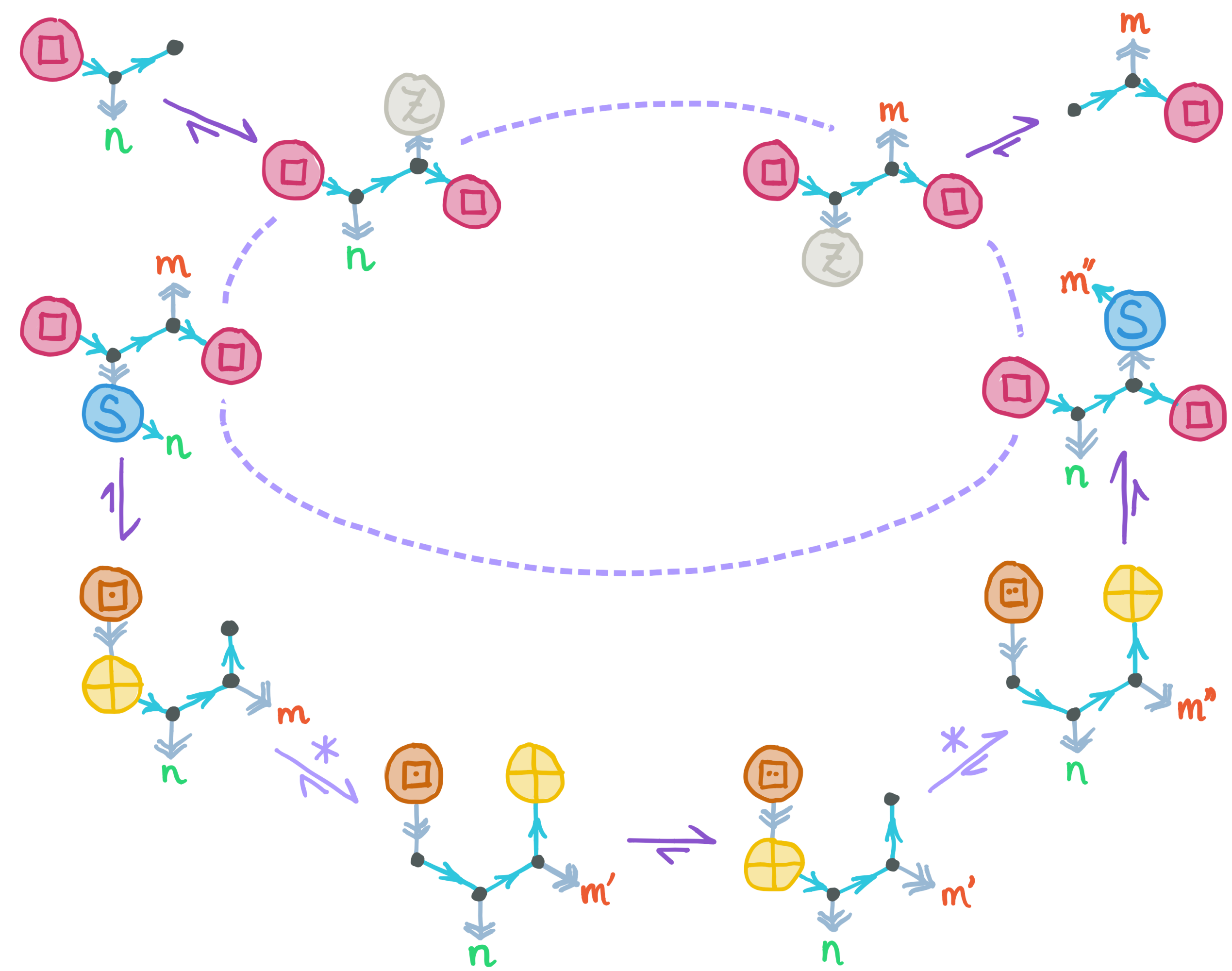
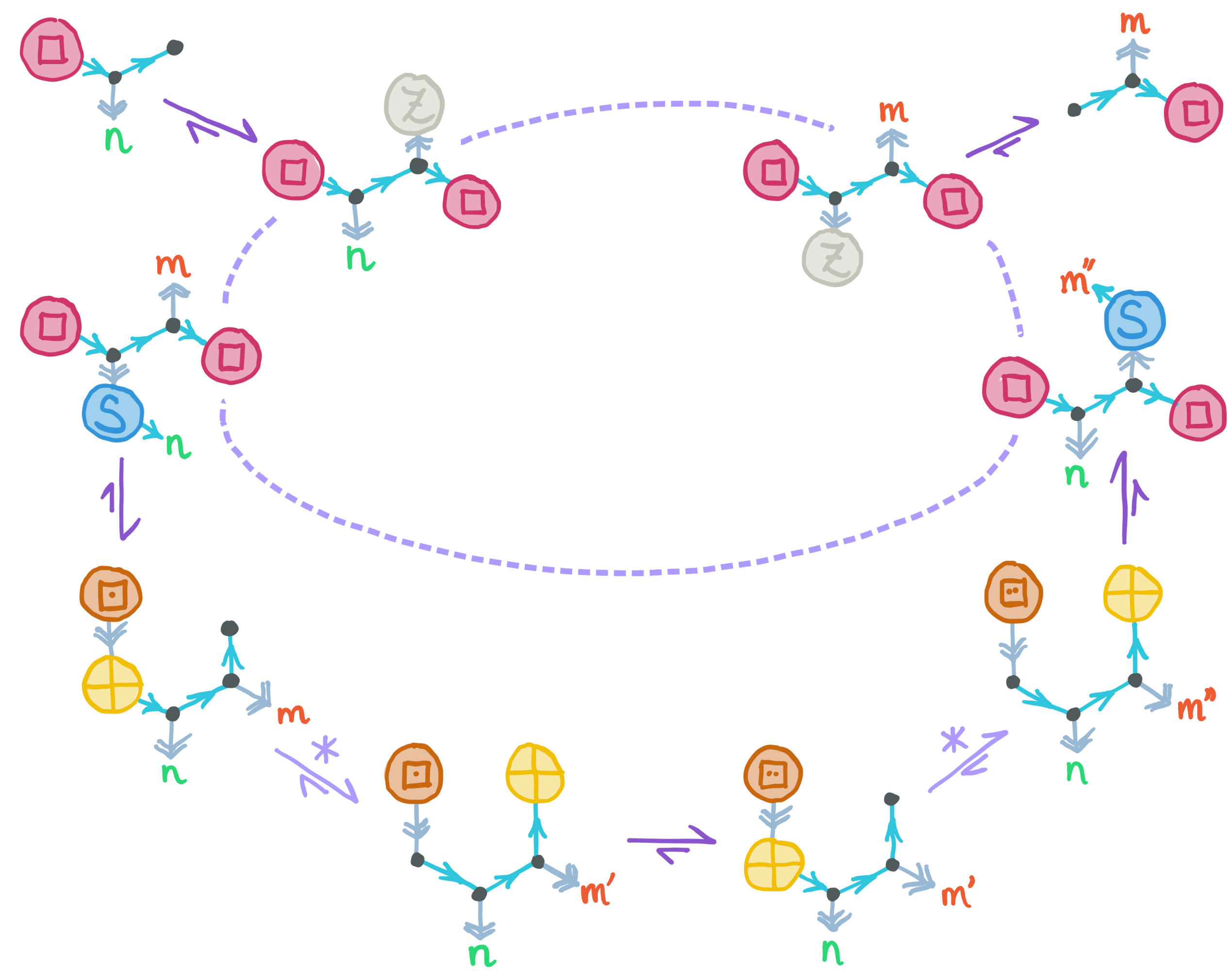
\quad \textsc{Sq}\ s\ (\mathrm{S}k)\ \textsc{Sq} = \textsc{Sq}\ (\mathrm{S}s'')\ k\ \textsc{Sq}: & (\textsc{sq–step}) \\
\qquad\quad +\ s\ k\ () = ()\ s'\ k\ +. & \text{-- } s' \leftarrow s\ + k \\
\qquad\quad +\ s'\ k\ () = ()\ s''\ k\ +. & \text{-- } s'' \leftarrow s' + k \\
\quad \textsc{Sq}\ n\ \mathrm{Z}\ \textsc{Sq} = ()\ n\ \textsc{Sq}; & (\textsc{sq–end})
\end{array}$$



$$\textbf{!}\quad ()\ 10\ \textsc{Sq} = \textsc{Sq}\ 10\ \mathrm{Z}\ \textsc{Sq} = \textsc{Sq}\ 9\ 1\ \textsc{Sq} = \textsc{Sq}\ 6\ 2\ \textsc{Sq} = \textsc{Sq}\ 1\ 3\ \textsc{Sq}$$

Squaring

$$m^2 = \sum_{k=0}^{m-1} (k + k + 1)$$



$\textbf{!}\ \textsc{Sq}\ m\ ();\quad \textbf{!}\ ()\ n\ \textsc{Sq};$

$\textsc{Sq}\ m\ () = \textsc{Sq}\ \mathrm{Z}\ m\ \textsc{Sq};$	(SQ–BEGIN)
$\textsc{Sq}\ s\ (\mathrm{S}k)\ \textsc{Sq} = \textsc{Sq}\ (\mathrm{S}s'')\ k\ \textsc{Sq}:$	(SQ–STEP)
$\quad + \ s\ k\ () = ()\ s'\ k\ +.$	-- $s' \leftarrow s + k$
$\quad + \ s'\ k\ () = ()\ s''\ k\ +.$	-- $s'' \leftarrow s' + k$
$\textsc{Sq}\ n\ \mathrm{Z}\ \textsc{Sq} = ()\ n\ \textsc{Sq};$	(SQ–END)

$\textbf{!}\quad ()\ 10\ \textsc{Sq} = \textsc{Sq}\ 10\ \mathrm{Z}\ \textsc{Sq} = \textsc{Sq}\ 9\ 1\ \textsc{Sq} = \textsc{Sq}\ 6\ 2\ \textsc{Sq} = \textsc{Sq}\ 1\ 3\ \textsc{Sq} \quad \bot$

Properties + Future Work

- r-Turing Complete
- Confluent Semantics
- Concurrent variant
- Interpreter written

- Implement & study concurrent variant
- Type system
- Apply to molecular programming

Alethe

%

```
a   Z   +   a     Z;
a (S b) + (S c) (S b):
    a b + c b.
```

%

```
n ^2 n2:
  ! Go n     Z = Go Z n2.
    Go (S n) m = Go n (S k):
        m n + l n.
        l n + k n.
```

%

```
a b `Pair` n:
  ! Go   n     Z   Z = Go Z   a    b.
    Go (S n)   Z   b = Go n (S b)  Z;
    Go (S n) (S a) b = Go n   a  (S b);
```

Thank you!

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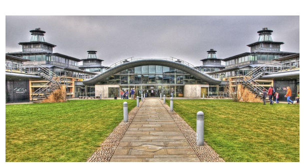

Department of Applied Mathematics and Theoretical Physics (DAMTP)

Hannah Earley — 2016–2021 — 'Modelling approaches to molecular computation' — EPSRC Project Reference 1781682
Hannah Earley — 2020/2022 — 'The $\aleph$-Calculus' — arXiv/Proceedings of RC22